สมบัติบางประการของปริภูมิใหญ่สุดย่อยแบบ sg ในปริภูมิโครงสร้างเล็กสุด

Some properties of sg-submaximal in minimal structure space

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บทคัดย่อ

ในบทความนี้เราศึกษาคุณสมบัติของ เซตเปิด เซตปิด ตัวดำเนินการส่วนปิดคลุม ตัวดำเนินการภายใน บนปริภูมิโครงสร้างเล็กสุด เราได้ให้ลักษณะของปริภูมิใหญ่สุดย่อยแบบ sg โดยใช้เซตปิดและเซตเปิดชนิดต่าง ๆ

คำสำคัญ : หนาแน่นแบบ m_X, ไม่หนาแน่นแบบ m_X, ปริภูมิใหญ่สุดย่อยแบบ sg

Abstract

In this paper, we study the properties of open sets, closed sets, closure operator and interior operator on minimal structure space. We will provide characterization of sg-submaximal space by using various kinds of generalized closed sets and open sets.

Keywords : m_X-dense , m_X-codense, sg-Submaximal

Introduction

In the literature[1], Maki introduced the notion of minimal structure. Also Popa and Noiri[2], introduced the notion of m_X-open sets, m_X-closed sets and then characterized those sets using m_X-closure and m_X-interior operators, respectively. After that Popa and Noiri[2] and Cao et al.[3], defined some new types of open sets and closed sets in topological space and obtained some results in topological space. Late[4], Rosas introduced some new types of open set and closed set in minimal structure. The concept of relationships of generalized closed sets and some new character-izations of sg-submaximal were introduced by Gansteer[5]. In this paper, we study the minimal structure space and properties of open set, closed set, closure and interior in this space, including the relationship between every type of closed set. We provide the characterization of sg-submaximal space.

Preliminaries

First, we recall some concepts and definitions which are useful in the results.

Definition 2.1[1] Let X be a non-empty set and $P(X)$ the power set of X A subfamily m_X of $P(X)$ is called a *minimal structure* (briefly $m-structure$) on X if m_X contains $\varnothing$ and X The pair (X,m_X) is called an $m-space$. Each member of m_X is said to be m_X-open set and the complement of m_X-open set is said to be m_X-closed set.

Definition 2.2[2] Let X be a non-empty set and m_X be an $m-structure$ on X For a subset A of X the m_X-closure of A denoted by $mCl(A)$ and the m_X-interior of A denoted by $mInt(A)$ are defined as follows:

(1) $mCl(A)=\bigcap\{B\subseteq X : X-B\in m_X \text{ and } A\subseteq B\}$,

(2) $mInt(A)=\bigcup\{B\subseteq X : B\in m_X \text{ and } B\subseteq A\}$.

Lemma 2.3[2] Let X be a non-empty set and m_X be an $m-structure$ on X. For $A,B\subseteq X$ the following

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statements hold:

(1) $mInt(A) \subseteq A$. If $A \in m_X$, then $mInt(A) = A$.

(2) $A \subseteq mCl(A)$. If $X - A \in m_X$, then $mCl(A) = A$.

(3) If $A \subseteq B$, then $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$.

(4) $mInt(A \cap B) \subseteq mInt(A) \cap mInt(B)$ and $mCl(A) \cup mCl(B) \subseteq mCl(A \cup B)$.

(5) $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(A)) = mCl(A)$.

(6) $X - mCl(A) = mInt(X - A)$ and $X - mInt(A) = mCl(X - A)$.

(7) $mCl(\varnothing) = \varnothing$, $mCl(X) = X$, $mInt(\varnothing) = \varnothing$ and $mInt(X) = X$.

Definition 2.4[6,7] Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is called:

(1) m_X-semi open set if $A \subseteq mCl(mInt(A))$,

(2) m_X-pre open set if $A \subseteq mInt(mCl(A))$,

(3) m_X-b open set if $A \subseteq mInt(mCl(A)) \cup mCl(mInt(A))$,

(4) m_X-α open set if $A \subseteq mInt(mCl(mInt(A)))$,

(5) m_X-regular open set if $A = mInt(mCl(A))$.

The complement of an m_X-semi open (resp. m_X-preopen, m_X-b open, m_X-α open, m_X-regular open) set is called an m_X-semi closed (resp. m_X-pre closed, m_X-b closed, m_X-α closed, m_X-regular closed) set. The collection of all m_X -semi open (resp. m_X-preopen, m_X-b open, m_X-α open, m_X-regular open) sets of X is denoted by $m_X SO(X)$ $(resp.\ m_X PO(X), m_X BO(X), m_X \alpha O(X), m_X RO(X))$.

Definition 2.5[6,7] Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is called:

(1) $sCl(A) = \bigcap \{B \subseteq X : B$ is a m_X-semi closed set and $A \subseteq B\}$,

(2) $pCl(A) = \bigcap \{B \subseteq X : B$ is a m_X-pre closed set and $A \subseteq B\}$,

(3) $bCl(A) = \bigcap \{B \subseteq X : B$ is a m_X-b closed set and $A \subseteq B\}$.

Definition 2.6[6,7] Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is called:

(1) $m_X - gb$ closed if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X$,

(2) $m_X - sg$ closed if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X SO(X)$,

(3) $m_X - gs$ closed if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X$,

(4) $m_X - gp$ closed if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X$.

The complement of an $m_X - gb$ closed (resp. $m_X - sg$ closed, $m_X - gs$ closed, $m_X - gp$ clos-ed) set is called an $m_X - gb$ open (resp. $m_X - sg$ open, $m_X - gs$ open, $m_X - gp$ open) set.

Lemma 2.7 [4] Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is called:

(1) $sCl(A) = A \cup mInt(mCl(A))$,

(2) $pCl(A) = A \cup mCl(mInt(A))$.

Definition 2.8[8] Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is called m_X-nowhere dense if and only if $mInt(mCl(A)) = \varnothing$.

Definition 2.9[8] Let (X, m_X) be an $m-space$ and $D \subseteq X$. Then D is called m_X-dense if and only if $mCl(D) = X$.

Definition 2.10[4] Let (X, m_X) be an $m-space$ and let $X_1, X_2 \subseteq X$ defined by X_1 $\{x \in X : \{x\}$ is $m_X - nowhere\ dense\}$ and $X_2 = \{x \in X : \{x\}$ is m_X -preopen}, It is easy to see that $\{X_1, X_2\}$ is a decomposition of X (i.e. $X = X_1 \cup X_2$).

We will give the definition of m_X-codense and $m_X - sg$ closed, including study intersection and relation-ships of some types of closed set.

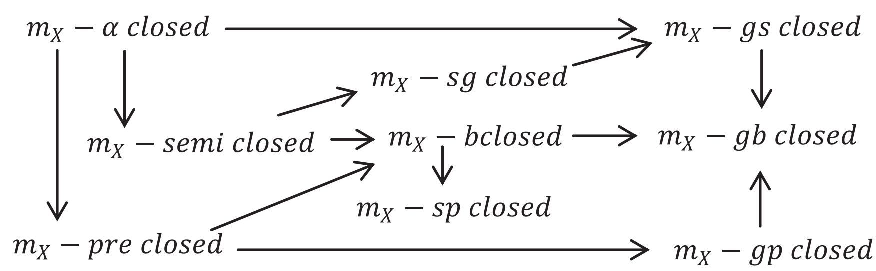
Definition 2.11 Let (X, m_X) be an $m-space$ and $E \subseteq X$. Then E is called m_X-codense if and only if $mInt(E) = \varnothing$.

Definition 2.12 An $m-space$ (X, m_X) is said to be sg-submaximal if every m_X-codense subset of X is $m_X - sg$ closed.

Example 2.13 Let $X = \{a, b, c\}$ Define the m-structure on X by $m_X = \{\varnothing, \{c\}, \{a, b\}, X\}$. Then $\varnothing$, $\{a\}, \{b\}$are m_X-codense. Moreover, we get $m_X SO(X)$ $\{\varnothing, \{c\}, \{a, b\}, X\}$.So $\varnothing$,$\{a\}, \{b\}$ are $m_X - sg$ closed. Hence (X, m_X) is sg-submaximal of X.

It is not difficult to prove that the intersect-ion of m_X-b closed (resp. m_X-semi closed, m_X-pre closed) is also m_X-b closed (resp. m_X-semi closed,m_X-preclosed).

The relationships between various types of generalized closed set have been summarized in the following diagram.



3. Results

First we will give a characterization of $m_X - sg$ closed in $m-space$.

Theorem 3.1 Let (X, m_X) be an $m-space$ and $A \subseteq X$. Then A is $m_X - sg$ closed if and only if $X_1 \cap sCl(A) \subseteq A$.

Proof. $(\Rightarrow)$ Let $x \in X_1 \cap sCl(A)$,, then $\{x\}$ is an $m-space$. Assume that $x \notin A$ then $A \subseteq X - \{x\}$. Thus $sCl(A) \subseteq X - \{x\}$, a contradict-ion. Therefore $x \in A$ that is $X_1 \cap sCl(A) \subseteq A$. $(\Leftarrow)$ Suppose that $X_1 \cap sCl(A) \subseteq A$. Let $U \in m_X SO(X)$ such that $A \subseteq U$ and let $x \in sCl(A)$. If $x \in X_1$ then $x \in X_1 \cap sCl(A) \subseteq A$. So $sCl(A) \subseteq A$. Assume now $x \in X_2$. Suppose that $x \notin U$. This implies that $X - U$ is m_X-semi closed and $x \in X - U$. Since $\{x\}$ is m_X-pre open, we have

$$\begin{aligned} sCl(\{x\}) &= \{x\} \cup mInt\ (mCl(\{x\})) \\ &= mInt(mCl(\{x\})) \\ &\subseteq mInt(mCl(X-U)) \\ &\subseteq X - U. \end{aligned}$$

Since $\{x\}$ is m_X-preopen and we get that $mInt(mCl(\{x\})) \cap A \neq \emptyset$, then let $y \in mInt(mCl(\{x\})) \cap A$, we get that $y \in mInt(mCl(\{x\})) \cap A \subseteq (X-U) \cap U = \emptyset$,contradiction. Thus $x \in U$ and $sCl(A) \subseteq U$. Hence A is m_X-sgclosed.

Lemma 3.2 If A is m_X-regular open and $mInt(A)$ is m_X-open, then A is m_X- open.

Proof. Let A be m_X-regular open, then $A = mInt(mCl(A))$. Thus

$$\begin{aligned} mInt(A) &= mInt\ (mInt(mCl(A))) \\ &= mInt(mCl(A)) = A. \end{aligned}$$

It implies that A is m_X - open.

It implies that A is m_X- open.

Lemma 3.3 If A is an $m_X - sg$ closed set and let B be an m_X-closed sets, then $A \cup B$ is $m_X - sg$ closed.

Proof. Let A be an $m_X - sg$ closed set and let B be an m_X-closed set. Then $X_1 \cap sCl(A) \subseteq A$. Consider,

$$\begin{aligned} X_1 \cap sCl(A \cup B) &\subseteq X_1 \cap (sCl(A) \cup sCl(B)) \\ &= sCl(A) \cup (X_1 \cap sCl(B)) \\ &= (A \cup mInt(mCl(A))) \\ &\quad \cup (X_1 \cap sCl(B)) \\ &= A \cup B \end{aligned}$$

therefore by Theorem 3.1, $A \cup B$ is $m_X - sg$ closed set.

Lemma 3.4 Let (X, m_X) be an $m-space$ and $A, B \subseteq X$. If A is an m_X-semi closed set and B is an $m_X - sg$ closed set, then $A \cap B$ is $m_X - sg$ closed set.

Proof. Let A be an m_X-semi closed set and B is $m_X - sg$ closed set, then $mInt(mCl(A)) \subseteq A$ and $X_1 \cap sCl(A) \subseteq A$. Consider,

$$\begin{aligned} X_1 \cap sCl(A \cap B) &\subseteq X_1 \cap (sCl(A) \cap sCl(B)) \\ &= sCl(A) \cap (X_1 \cap sCl(B)) \\ &= (A \cup mInt(mCl(A))) \\ &\quad \cap (X_1 \cap sCl(B)) \\ &= A \cap B, \end{aligned}$$

therefore by Theorem 3.1,$A \cap B$ is $m_X - sg$ closed set.

Lemma 3.5 Let (X, m_X) be an $m-space$ and $A \in X$. Then $bCl(A) = sCl(A) \cap pCl(A)$.

Proof. Consider, $bCl(A)$

$$\begin{aligned} &= A \cup (mInt(mCl(A)) \cap mCl(mInt(A))) \\ &= (A \cup mInt(mCl(A))) \cap \\ &(A \cup mCl(mInt(A))), \end{aligned}$$

by Lemma 2.7, $bCl(A) = sCl(A) \cap pCl(A).$

We now consider the property of sg-submaximal. First we will give some elementary characterizations of sg-submaximal spaces.

Theorem 3.6 Let X be an $m-space,$ the following properties are equivalent:

(1) X is sg-submaximal,

(2) For any subset A of X, $A = mCl(A) \cap G$ where G is an $m_X - sg$ open subset of X,

(3) For any subset A of X, $A = mInt(A) \cup F$ where F is an $m_X - sg$ closed subset of X,

(4) every m_X-codense subset A of X is $m_X - sg$ closed,

(5) $mCl(A) - A$ is $m_X - sg$ closed for every subset A of X.

Proof. (1) $\Rightarrow$(2): Let $A \subseteq X$. We consider

$mInt(mCl(A) - A)$
$= mInt(mCl(A) \cap (X - A))$
$\subseteq mInt(mCl(A)) \cap mInt(X - A)$
$= mInt(mCl(A)) \cap [X - mCl(A)]$
$\subseteq mCl(A) \cap [X - mCl(A)] = \varnothing.$

This implies that $mCl(A) - A$ is m_X - codense. By (1) we get $mCl(A) - A$ is $m_X - sg$ closed. Then $(X - mCl(A)) \cup A$
$= X - (mCl(A) \cap (X - A)) = X - (mCl(A) - A)$
is $m_X - sg$ open. Therefore
$[(X - mCl(A)) \cup A] \cap mCl(A)$
$= [(X - mCl(A)) \cap mCl(A)] \cup [A \cap mCl(A)]$
$= A.$ Hence we can conclude that (2) is true.

(2)$\Rightarrow$(3): Let $A \subseteq X$. Then there exists an $m_X - sg$ open subset G of X such that $X - A = mCl(X - A) \cap G.$ Thus
$A = X - [X - A]$
$= X - [mCl(X - A) \cap G]$
$= (X - mCl(X - A)) \cup (X - G)$
$= mInt(A) \cup (X - G).$ This implies that
$= mInt(A) \cup (X - G).$ This implies that
$X - G$ is an $m_X - sg$ closed subset of X.
Hence the statement (3) is true.

(3)$\Rightarrow$(4): Let A be m_X-codense, that is $mInt(A) = \varnothing.$ By (3), there exists an $m_X - sg$ closed subset F of X such that $A = mInt(A)$ $\cup F.$ Hence $A = mInt(A) \cup F = \varnothing \cup F = F.$ So A is $m_X - sg$ closed.

(4)$\Rightarrow$(5): Let $A \subseteq X$. We consider,
$mInt(mCl(A) - A)$
$= mInt(mCl(A) \cap (X - A))$
$\subseteq mIn(mCl(A)) \cap mInt(X - A)$
$= mInt(mCl(A)) \cap [X - mCl(A)]$
$\subseteq mCl(A) \cap [X - mCl(A)] = \varnothing.$
This implies that $mCl(A) - A$ is m_X-codense, therefore $mCl(A) - A$ is $m_X - sg$ closed.

(5)$\Rightarrow$(1): Let A be m_X-codense of X, that is $mInt(A) = \varnothing.$ By (5), we get that $mCl(X - A)$ $-(X - A)$ is $m_X - sg$ closed. We also have that
$mCl(X - A) - (X - A) = mCl(X - A) \cap A$
$= [X - mInt(A)] \cap A = X \cap A = A.$
Hence A is $m_X - sg$ closed. Therefore X is sg-submaximal.

Example 3.7 Let $X = \{a, b, c\}$. Define the m-structure on X by $m_X = \{\varnothing, \{a\}, \{a, b\}, X\}.$

Then $\varnothing, \{c\}, \{b, c\}$ are m_X-codense. Moreover, we get $m_X SO(X) = \{\varnothing, \{a\}, \{a, b\}, \{a, c\}, X\}.$ So $\varnothing, \{c\}, \{b, c\}$ are $m_X - sg$ closed. Hence (X, m_X) is sg-submaximal of X. It is clear that (1) and (4) are equivalent. For (2), (3), (5) it is not difficult to show how they are equivalent.

Theorem 3.8 Let X be an $m-space,$ and let $mInt(E)$ be an open set when $E \subseteq X$, the following properties are equivalent:

(1) every m_X-b closed set is $m_X - sg$ closed,

(2) every m_X-pre closed set is $m_X - sg$ closed,

(3) X is sg-submaximal.

Proof. (1)$\Rightarrow$(2): Let A be m_X-pre closed, that is $mCl(mInt(A)) \subseteq A.$ Then
$mCl(mInt(A)) \cap mInt(mCl(A))$
$\subseteq mCl(mInt(A)) \cap mCl(mCl(A))$
$= mCl(mInt(A)) \cap mCl(A)$
$\subseteq A \cap mCl(A) = A.$

This implies that A is m_X-b closed, therefore A is $m_X - sg$ closed.

(2)$\Rightarrow$(1): Let A be m_X -b closed, then $A = bCl(A)$. By Lemma 3.5, we get $bCl(A) = sCl(A) \cap pCl(A)$. We can easily see that $sCl(A)$ is m_X -semi closed and $pCl(A)$ is m_X -pre closed. Therefore $pCl(A)$ is $m_X - sg$ closed. By Lemma 3.4, implies that $A = bCl(A) = sCl(A) \cap pCl(A)$. Hence A is $m_X - sg$ closed.

(2)$\Rightarrow$(3): Let A be m_X -codense, then $mInt(A) = \varnothing$. Since $mCl(mInt(A)) = mCl(A) = \varnothing \subseteq A$. Thus $mCl(mInt(A)) \subseteq A$, such that A is m_X -pre closed. Therefore A is $m_X - sg$ closed. Hence X is sg -submaximal.

(3)$\Rightarrow$(2): Let A be m_X -pre closed, than $X - A$ is m_X -preopen and we will get

$X - A \subseteq mInt(mCl(X - A))$.

Let $G = mInt(mCl(X - A))$. Then we get $mCl(X - A) \subseteq mCl(G)$. Consider $mCl(G) = mCl(mInt(mCl(X - A))) \subseteq mCl(X - A)$. Thus $mCl(G) = mCl(X - A)$. This implies that $G = mInt(mCl(G))$, i.e. G is m_X -regular open. Since $mCl(G) \subseteq X$, then $mInt(G)$ is open set. By Lemma 3.2, G is an open set. Assume that $D = (X - A) \cup (X - G)$, then

$$\begin{aligned} mCl(D) &= mCl[(X - A) \cup (X - G)] \\ &= mCl(X - A) \cup mCl(X - G) \\ &= mCl(G) \cup X - G \\ &= mCl(X) = X, \end{aligned}$$

therefore D is m_X -dense. Consider,

$$\begin{aligned} D \cap G &= [(X - A) \cup (A - G)] \cap G \\ &= [(X - A) \cap G] \cup [(X - G) \cap G] \\ &= [(X - A) \cap G] \cup \varnothing = X - A, \end{aligned}$$

thus $A = (X - D) \cup (X - G)$. Consider $X - D$ we will get $mInt(X - D) = X - mCl(D) = X - X = \varnothing$, thus $X - D$ is m_X -codense. Since X is sg -submaximal, then $X - D$ is $m_X - sg$ closed. Since $X - G$ is a closed set and by Lemma 3.3, $A = (X - D) \cup (X - G)$ is $m_X - sg$ closed.

Example 3.9 Let $X = \{a,b,c\}$. Define the m-structure on X by $m_X = \{\varnothing, \{a\}, \{b\}, X\}$. Then $\varnothing, \{c\}$ are m_X -codense. Moreover, we can find that $m_X SC(X)$

$= \{\varnothing, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$,

$m_X BC(X)$

$= \{\varnothing, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$ and

$m_X PC(X) = \{\varnothing, \{a\}, \{b\}, \{a,b\}, X\}$. So $\varnothing$, $\{c\}$ are $m_X - sg$ closed. Hence (X, m_X) is sg -submaximal of X and (1)-(3) are equivalent.

Conclusion

In conclusion, the concepts of minimal structure space which study open set, closed set, closure and interior in intersects on such. The results are properties characterizations of Sg-submaximal space in Theorem 3.6 and Theorem 3.8

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