เซต $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด

$\mathcal{A}$-Sets in Biminimal Structure Spaces

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บทคัดย่อ

งานวิจัยนี้จะนำเสนอแนวคิดเกี่ยวกับเซต $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุดและตรวจสอบคุณสมบัติบางประการของเซต $\mathcal{A}$ รวมทั้งฟังก์ชันต่อเนื่อง $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด

คำสำคัญ : เซต $\mathcal{A}$, ฟังก์ชันต่อเนื่อง $\mathcal{A}$

Abstract

In this article, we introduce the concepts of $\mathcal{A}$-sets in biminimal structure spaces and investigate some of their properties. Moreover, the notions of $\mathcal{A}$-sets and $\mathcal{A}$-continuous functions in biminimal structure spaces were studied.

Keywords : $\mathcal{A}$-set, $\mathcal{A}$-continuous function.

Introduction

In 1972, J. Dugundji[7] introduced the concepts of regular closed sets in topological spaces. Let (X,τ) be a topological space and $A\subseteq X$, then A is called regular closed if and only if $A=Cl(Int(A))$. In 1986, J.Tong[21] introduced the concepts and properties of $\mathcal{A}$-sets in topological spaces. Let A be a subset of a topological space (X,τ) then A is an $\mathcal{A}$-set in (X,τ) if $A=U\cap B$ when U is open and B is regular closed in (X,τ). In addition, J.Tong[21] introduced the concepts of $\mathcal{A}$-continuous functions from a topological space (X,τ) to a topological space $(Y,\mathcal{U})$. Let f be a function from X to Y, then f is $\mathcal{A}$-continuous function if and only if the inverse image of each open set in Y is an $\mathcal{A}$-set in X. In 1990, M. Ganster, and Reilly, I. L.[10] improved J. Tong's decomposition result and provided a decomposition of $\mathcal{A}$-continuous. In 2000, the concepts of minimal structure spaces were introduced by V. Popa and T. Noiri[18]. A pair (X, m_X) is a minimal structure space if and only if $X \neq \emptyset$ and m_X is family of $P(X)$ with $\emptyset, X\in m_X$. Moreover, they also introduced the concepts of m_X-open sets and m_X-closed sets in minimal structure spaces. In 1963, J. C. Kelly[9] introduced the concepts of bitopological spaces which consist of a nonempty set and two topological spaces. In 2010, C. Boonpok[2] introduced the concepts of biminimal structure spaces which consist of a nonempty set and two minimal structures. Furthermore, C. Boonpok[2] defined $m_X^1m_X^2$-closed sets in biminimal structure spaces and the complement of $m_X^1m_X^2$-closed sets is call $m_X^1m_X^2$-open sets. In 2010, C. Boonpok [4] defined (i, j) m_X-regular open sets in biminimal structure spaces and he also defined (i, j) m_X-regular closed sets as complement of (i,j) m_X-regular open sets for $i, j = 1, 2$ and $i\neq j$.

In this article we introduce the concepts of $\mathcal{A}$-sets in biminimal structure spaces and $\mathcal{A}$-continuous functions in biminimal structure spaces. Also, we study some properties of $\mathcal{A}$-sets and $\mathcal{A}$-continuous functions in biminimal structure spaces.

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Preliminaries

In this section, we will give some definitions and notations, deal with some preliminaries and some useful results that will be duplicated in later sections.

Definition 2.1[10] Let (X, τ) be a topological space and $M \subseteq X$. Then M is called an $\mathcal{A}$-set if $M = U \cap B$ when U is open and B is regular closed in X.

The family of all $\mathcal{A}$-sets in a topological space (X, τ) is denoted by $\mathcal{A}(X, \tau)$.

Definition 2.2[17] Let X be a nonempty set and $P(X)$ be the power set of X. A subfamily m_x of $P(X)$ is called a minimal structure (briefly an m-structure) on X if $\varnothing \in m_x$ and $X \in m_x$.

The pair (X, m_x), we denote a nonempty set X with an m-structure m_x on X and it is called a minimal structure space (briefly an m-space). Each member of m_x is said to be m_x-open and the complement of an m_x-open set is said to be m_x-closed.

Definition 2.3[17] Let X be a nonempty set and m_x an m-structure on X. For a subset A of X the m_x-interior of A and the m_x-closure of A with respect to m_x are defined as follows:

$m_xInt(A) = \bigcup\{ U : U \subseteq A,\ U \in m_x\}$,

$m_xCl(A) = \bigcap\{ F : A \subseteq F,\ X \setminus F \in m_x \}$.

Lemma 2.4[14] Let X be a nonempty set and m_x an m-structure on X. For any subsets A and B of X, the following properties hold:

(1) $m_xCl(X \setminus A) = X \setminus m_xInt(A)$ and $m_xInt(X \setminus A) = X \setminus m_xCl(A)$,

(2) If $(X \setminus A) \in m_x$, then $m_xCl(A) = A$ and if $A \in m_x$, then $m_xInt(A) = A$,

(3) $m_xCl(\varnothing) = \varnothing$, $m_xCl(X) = X$, $m_xInt(\varnothing) = \varnothing$ and $m_xInt(X) = X$,

(4) If $A \subseteq B$, then $m_xCl(A) \subseteq m_xCl(B)$ and $m_xInt(A) \subseteq m_xInt(B)$,

(5) $A \subseteq m_xCl(A)$ and $m_xInt(A) \subseteq A$,

(6) $m_xCl(m_xCl(A)) = m_xCl(A)$ and $m_xInt(m_xInt(A)) = m_xInt(A)$,

(7) $m_xInt(A \cap B) = m_xInt(A) \cap m_xInt(B)$ and $m_xInt(A) \cup m_xInt(B) \subseteq m_xInt(A \cup B)$,

(8) $m_xCl(A \cup B) = m_xCl(A) \cup m_xCl(B)$ and $m_xCl(A \cap B) \subseteq m_xCl(A) \cap m_xCl(B)$.

Definition 2.5[13] An m-structure m_x on a nonempty set A is said to have property $\mathfrak{B}$ if the union of any family of subsets belonging to m_x belongs to m_x.

Lemma 2.6[17] Let X be a nonempty set and m_x is an m-structure on X satisfying property $\mathfrak{B}$.

For $A \subseteq X$ the following properties hold:

(1) $A \in m_x$ if and only if $m_xInt(A) = A$,

(2) A is m_x-closed if and only if $m_xCl(A) = A$,

(3) $m_xInt(A)$ is m_x-open and $m_xCl(A)$ is m_x-closed.

Definition 2.7[3] Let (X, m_x) be an m-space and $R \subseteq X$. Then R is called m_x-regular closed if and only if $R = m_xCl(m_xInt(R))$.

The family of all m_x-regular closed sets in an m_x-space (X, m_x) is denoted by $RC(X, m_x)$

Definition 2.8[19] A subset A of an m-space (X, m_x) is called an m-preopen set if $A \subseteq m_xInt(m_xCl(A))$ and an m_x-preclosed set if $m_xCl(m_xInt(A)) \subseteq A$.

The family of all m_x-preopen sets in an m- space (X, m_x) is denoted by $PO(X, m_x)$ and m preclosed sets in an m-space (X, m_x) is denoted by $PC(X, m_x)$

Definition 2.9[19] Let (X, m_x) be an m-space and $A \subseteq X$, the m_x-preclosure of A is denoted by $m_xpcl(A)$ is defined as the intersection of all m_x-preclosed of (X, m_x) containing A.

Proposition 2.10[19] Let (X, m_x) be an m-space and $A, B \subseteq X$. If $A \subseteq B$ then $m_xpcl(A) \subseteq m_xpcl(B)$.

Proposition 2.11[19] Let (X, m_x) be an m-space and $A \subseteq X$. If m_x satisfies the property $\mathfrak{B}$. Then $m_xpcl(A) = A \cup m_xCl(m_xInt(A))$.

Definition 2.12[2] Let A be a nonempty set and $m_x^1 m_x^2$ be m-structures on X. A triple (X, m_x^1, m_x^2) is called a bi-minimal structure space (briefly *bim*- space).

Let (X, m_x^1, m_x^2) be a biminimal structure space and $A \subseteq X$. The m_x-closure and m_x-interior of A with respect to m_x^i are denoted by $m_xCl(A)$ and $m_xInt(A)$ respectively, for $i, j = 1, 2$.

Each member of m_x^i is said to be an m_x^i-open set and the complement of an open set is said to be m_x^i-closed, for $i, j = 1, 2$.

Definition 2.13[4] A subset A of biminimal structure spaces (X, m_X^1, m_X^2) is said to be

(1) $(i, j)m_X$-regular open if $A = m_X^i Int(m_X^j Cl(A))$, where i, j = 1 or 2 and $i \neq j$,

(2) $(i, j)m_X$-semi-open if $A \subseteq m_X^i Cl(m_X^j Int(A))$ where i, j = 1 or 2 and $i \neq j$,

(3) $(i, j)m_X$-preopen if $A \subseteq m_X^i Int(m_X^j Cl(A))$, where i, j = 1 or 2 and $i \neq j$.

The complement of an $(i, j)m_X$-regular open (resp. ($(i, j)m_X$-semi-open, $(i, j)m_X$-preopen) set is called $(i, j)m_X$-regular closed (resp,($(i, j)m_X$-semi-closed, $(i, j)m_X$-preclosed).

Lemma 2.14[4] Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X. Then

(1) A is $(i, j)m_X$-regular closed if and only if $A = m_X^i Cl(m_X^j Int(A))$,

(2) A is $(i, j)m_X$-semi-closed if and only if $m_X^i Int(m_X^j Cl(A)) \subseteq A$,

(3) A is $(i, j)m_X$-preclosed if and only if $m_X^i Cl(m_X^j Int(A)) \subseteq A$.

Definition 2.15 [4] Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be (i, j)-M-continuous at a point $x \in X$ and each $V \in m_Y^i$ containing $f(X)$, there exists $U \in m_X^j$ containing x such that $f(U) \subseteq V$, where i, j = 1 or 2 and $i \neq j$.

A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be (i, j)-M-continuous if it has this property at each point $x \in X$.

Theorem 2.16[4] For a function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$, the following properties are equivalent:

(1) f is (i, j)-M-continuous;

(2) $f^{-1}(V) = m_X^j Int(f^{-1}(V))$ for every $V \in m_Y^i$

(3) $f(m_X^i Cl(A)) \subseteq m_Y^i Cl(f(A))$ for every subset A of X;

(4) $m_X^j Cl(f^{-1}(B)) \subseteq f^{-1}(m_Y^i Cl(B))$ for every subset B of Y;

(5) $f^{-1}(m_Y^i Int(B)) \subseteq m_X^j Int(f^{-1}(B))$ for every subset B of Y;

(6) $m_X^j Cl(f^{-1}(F)) = f^{-1}(F)$ for every m_Y^i-closed set F of Y.

Results and Discussion

$\mathcal{A}$-sets in minimal structure space

In this section, we introduce the concept of $\mathcal{A}$-sets in minimal structure spaces.

Definition 3.1.1 Let (X, m_X) be an m-space. A subset M of A is said to be an m_X-$\mathcal{A}$-set if there exist G and R such that $M = G \cap R$ when G is m_X-open and R is m_X-regular closed.

The family of all m_X-$\mathcal{A}$-set in an m-space (X, m_X) is denoted by $\mathcal{A}(X, m_X)$.

Example 3.1.2 Let $X = \{1, 2, 3\}$. Define an m-structure m_X on X as follows : $m_X = \{\emptyset, \{2\}, \{1, 2\}, \{1, 3\}, X\}$. Then $RC(X, m_X) = \{\emptyset, \{2\}, \{1, 3\}, X\}$ and $\mathcal{A}(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, X\}$.

Definition 3.1.3 Let (X, m_X) be an m-space and $A \subseteq X$, then A is said to be an m_X-t-set if $m_X Int(A) = m_X Int(m_X Cl(A))$.

The family of all m_X-t-set in an m-space (X, m_X) is denoted by $t(X, m_X)$.

Example 3.1.4 Let $X = \{1, 2, 3\}$ and define $m_X = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ be an m-structure on X. It follows that $t(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$.

Proposition 3.1.5 Let (X, m_X) be an m-space and $R \subseteq X$. If R is m_X-regular closed then R is an m_X-t-set.

Proof. Let R be an m_X-regular closed. Then $R = m_X Cl(m_X Int(R))$. Consequently, $m_X Cl(R) = m_X Cl(m_X Cl(m_X Int(R)))$. Thus $m_X Int(m_X Cl(R)) = m_X Int(m_X Cl(m_X Int(R)))$. Hence $m_X Int(m_X Cl(R)) = m_X Int(R)$. Therefore, R is an m_X-t-set.

$\mathcal{A}$-sets in biminimal structure space

In this section, we introduce the concept of $\mathcal{A}$-sets in biminimal structure spaces and study some fundamental properties of $\mathcal{A}$-sets in biminimal structure spaces and investigate some of their properties.

Definition 3.2.1 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be $(i, j)m_X$-locally closed if there exist G and F such that $A = G \cap F$ when G is an m_X^i-open set G and F is an m_X^j-closed set, where i, j = 1 or 2 and $i \neq j$.

The family of all $(i, j)m_X$-locally closed sets in biminimal structure spaces (X, m_X^1, m_X^2) is denoted by $(i, j)m_X$-$\mathcal{LC}(X, m_X^1, m_X^2)$, where i, j = 1 or 2 and $i \neq j$.

Example 3.2.2 Let $X = \{a, b, c\}$. Define m-structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{b, c\}, X\}$ and $m_X^2 = \{\emptyset, \{c\}, X\}$. Thus $(1, 2)$-$\mathcal{LC}(X, m_X^1, m_X^2) = \{\emptyset, \{b\}, \{b, c\}, \{a, b\}, X\}$.

Lemma 3.2.3 Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let $i, j = 1$ or 2 and $i \neq j$. If S is an $(i, j)m_X$-locally closed set then there exists an m_X^i-open set U such that $S = U \cap m_X^jCl(S)$

Proof. Let S be an $(i, j)m_X$-locally closed set. Then there exist U and F such that $S = U \cap F$ where U is m_X^i-open and F is m_X^j-closed. Since $S = U \cap F$, $S \subseteq F$. Thus $m_X^jCl(S) \subseteq m_X^jCl(F)$. Since F is m_X^j-closed, $m_X^jCl(S) \subseteq F$. Then $U \cap m_X^jCl(S) \subseteq U \cap F = S$. Since $S \subseteq U$ and $S \subseteq m_X^jCl(S)$. Then $S \subseteq U \cap m_X^jCl(S)$. Therefore, there exists an m_X^i-open set U such that $S = U \cap m_X^jCl(S)$.

The converse of Lemma 3.2.3 is true if m_X^j has property $\mathfrak{B}$ as a following proposition.

Proposition 3.2.4. Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let m_X^j has property $\mathfrak{B}$, where $i, j = 1$ or 2 and $i \neq j$. Then S is an $(i, j)m_X$-locally closed set iff there exists an m_X^i-open set U such that $S = U \cap m_X^jCl(S)$.

Proof. ($\Longrightarrow$) By Lemma 3.2.3.

($\Longleftarrow$) Let $S = U \cap m_X^jCl(S)$, for some $U \in m_X^i$. Since m_X^j has property $\mathfrak{B}$, $m_X^jCl(S)$ is closed in (X, m_X^j). Thus S is an $(i, j)m_X$-locally closed.

Definition 3.2.5. Let (X, m_X^1, m_X^2) be a biminimal structure space. A subset M of X is said to be an $(i, j)m_X$-$\mathcal{A}$-set if there exist G and R, such that $M = G \cap R$ when $G \in m_X^i$ and R is m_X^j-regular closed, where $i, j = 1$ or 2 and $i \neq j$.

The family of all $(i, j)m_X$-$\mathcal{A}$-sets in a biminimal structure space (X, m_X^1, m_X^2) is denoted by (i, j)-$\mathcal{A}(X, m_X^1, m_X^2)$ where $i, j = 1$ or 2 and $i \neq j$.

Example 3.2.6. Let $X = \{1, 2, 3\}$. Define m-structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1, 2\}, X\}$ which are m-structures on X. It follows that $RC(X, m_X^2) = \{\emptyset, X\}$. Thus $(1, 2)\mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$.

Lemma 3.2.7. Let (X, m_X^1, m_X^2) be a biminimal structure space m_X^j has property $\mathfrak{B}$. If a subset M of X is an $(i, j)m_X$-$\mathcal{A}$-set, then M is $(i, j)m_X$-locally closed, where $i, j = 1$ or 2 and $i \neq j$.

Proof. Let M is an $(i, j)m_X$-$\mathcal{A}$-set. Then there exist G and R such that $M = G \cap R$ where $G \in m_X^i$-open and R is m_X^j-regular closed. Since R is m_X^j-regular closed, $R = m_X^jCl(m_X^jInt(R))$. But m_X^j has property $\mathfrak{B}$, then $m_X^jCl(m_X^jInt(R))$ is closed. Hence R is m_X^j-closed. It follows that M is an $(i, j)m_X$-locally closed.

Proposition 3.2.8 Let (X, m_X^1, m_X^2) be a biminimal structure space and $m_X^j \subseteq m_X^i$ has the property $\mathfrak{B}$. If a subset M of X is both $(i, j)m_X$-semi-open and $(i, j)m_X$-locally closed, then M is an $(i, j)m_X$-$\mathcal{A}$-sets, where $i, j = 1$ or 2 and $i \neq j$.

Proof. Let M be both $(i, j)m_X$-semi-open and $(i, j)m_X$-locally closed. It follows that $M \subseteq m_X^jCl(m_X^iInt(M))$ such that $M = U \cap m_X^jCl(M)$. Since $m_X^jCl(M) \subseteq m_X^jCl(m_X^iInt(M))$. But $m_X^jCl(m_X^jInt(M)) \subseteq m_X^jCl(M)$, hence $m_X^jCl(M) = m_X^jCl(m_X^jInt(M))$ and $m_X^jCl(m_X^jInt(M))$ is m_X^j-regular closed. Consequently $m_X^jCl(M)$ is m_X^j-regular closed. Therefore, M is an $(i, j)m_X$-$\mathcal{A}$-set.

Definition 3.2.9 Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. Then A is said to be an $(i, j)m_X$-t-set if $m_X^iInt(A)) = m_X^iInt(m_X^jCl(A))$, where $i, j = 1$ or 2 and $i \neq j$.

The family of all $(i, j)m_X$-t-sets in a biminimal structure spaces (X, m_X^1, m_X^2) is denoted by (i, j)-$t(X, m_X^1, m_X^2)$ for $i, j = 1$ or 2 and $i \neq j$.

Example 3.2.10 Let $X = \{1, 2, 3\}$. Define m-structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, \{3\}, \{2, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{1, 2\}, X\}$.

Thus $(1, 2)$-$t(X, m_X^1, m_X^2) = \{\emptyset, \{3\}, \{2, 3\}, X\}$.

Theorem 3.2.11 Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. Then A is an $(i, j)m_X$-t-set if and only if A is an $(i, j)m_X$-semi-closed, where $i, j = 1$ or 2 and $i \neq j$.

Proof. ($\Longrightarrow$) Let A be an $(i, j)m_X$-t-set. Then $m_X^iInt(A)) = m_X^iInt(m_X^jCl(A))$. Thus $m_X^iInt(m_X^jCl(A)) \subseteq A$. Hence A is an $(i, j)m_X$-semi-closed.

($\Longleftarrow$) Let A be an $(i, j)m_X$-semi-closed. Then $m_X^iInt(m_X^jCl(A)) \subseteq A$. Thus $m_X^iInt(m_X^iInt(m_X^jCl(A))) \subseteq m_X^iInt(A)$. Hence $m_X^iInt(m_X^jCl(A)) \subseteq m_X^iInt(A)$. Since $m_X^iInt(A) \subseteq m_X^iInt(m_X^jCl(A))$. Thus $m_X^iInt(A) = m_X^iInt(m_X^jCl(A))$. Hence A is an $(i, j)m_X$-t-set.

Definition 3.2.12 Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. Then A is said to be an $(i, j)m_X$-$\mathcal{B}$-set if $A = U \cap T$, when U is an m_X^i-open set and T is an m_X^j-t-set, where $i, j = 1$ or 2 and $i \neq j$.

The family of all $(i, j)m_x$-$\mathcal{B}$-sets in a biminimal structure space (X, m_x^1, m_x^2) is denoted by (i, j)-$\mathcal{B}(X, m_x^1, m_x^2)$, where i, j = 1 or 2 and $i \neq j$.

Example 3.2.13 Let $X = \{1, 2, 3\}$. Define m-structures m_x^1 and m_x^2 on X as follows : $m_x^1 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$ and $m_x^2 = \{\emptyset, \{1\}, \{3\}, \{2, 3\}, X\}$. Then $\{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, X\}$ are m_x^2-$\mathcal{t}$-sets. Therefore, $(1, 2)$-$\mathcal{B}(X, m_x^1, m_x^2) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, X\}$.

Theorem 3.2.14 Let (X, m_x^1, m_x^2) be a biminimal structure space and $A \subseteq X$. If A is an $(i, j)m_x$-$\mathcal{A}$-set, then A is an $(i, j)m_x$-$\mathcal{B}$-set for all i, j = 1 or 2 and $i \neq j$.

Proof. Let A be an $(i, j)m_x$-$\mathcal{A}$-set. Then there exist G and R such that $A = G \cap R$ where G is m_x^i-open in (X, m_x^i) and R is an m_x^j-regular closed. By Proposition 3.1.5, R is an m_x^j-$\mathcal{t}$-set. Hence A is an $(i, j)m_x$-$\mathcal{B}$-set.

Proposition 3.2.15 Let (X, m_x^1, m_x^2) be a biminimal structure space and M be a subset of X. If M is an $(i, j)m_x$-locally closed set, then it is also an $(i, j)m_x$-$\mathcal{B}$-set, where i, j = 1 or 2 and $i \neq j$.

Proof. Let M be $(i, j)m_x$-locally closed set. Then there exist U and B such that $M = U \cap B$ where U is m_x^i-open and B is m_x^j-closed. Since B is m_x^j- closed, $B = m_x^jCl(B)$. Thus $m_x^jInt(B) = m_x^jInt(m_x^jCl(B))$. Hence B is an m_x^j-$\mathcal{t}$-set. Thus M is an $(i, j)m_x$-$\mathcal{B}$-set.

Definition 3.2.16 Let (X, m_x^1, m_x^2) be a biminimal structure space and $A \subseteq X$. Then A is said to be an $(i, j)m_x$-$\mathcal{C}$-set if $A = U \cap B$, when U is an m_x^i-open and B is m_x^j-preclosed, where i, j = 1 or 2 and $i \neq j$.

The family of all $(i, j)m_x$-$\mathcal{C}$-sets in a biminimal structure space (X, m_x^1, m_x^2) is denoted by (i, j)-$\mathcal{C}$ (X, m_x^1, m_x^2), where i, j = 1 or 2 and $i \neq j$.

Example 3.2.17 Let $X = \{1, 2, 3\}$. Define m-structures m_x^1 and m_x^2 on X as follows : $m_x^1 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$ and $m_x^2 = \{\emptyset, \{1\}, \{3\}, \{2, 3\}, X\}$. Thus $\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, X$ are preclosed in (X, m_x^2). Therefore, $(1, 2)$-$\mathcal{C}(X, m_x^1, m_x^2) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, X\}$.

Theorem 3.2.18. Let (X, m_x^1, m_x^2) be a biminimal structure space and $A \subseteq X$. If A is an $(i, j)m_x$-$\mathcal{A}$-set, then A is an $(i, j)m_x$-$\mathcal{C}$-set for all i, j = 1 or 2 and $i \neq j$.

Proof Let A be an $(i, j)m_x$-$\mathcal{A}$-set. Then there exist G and R such that $A = G \cap R$ where G is m_x^i-open and R is an m_x^j- regular closed. Since $R = m_x^jCl(m_x^jInt(R))$, thus $m_x^jCl(m_x^jInt(R)) \subseteq R$. Hence R is an m_x^j-preclosed. Therefore, A is an $(i, j)m_x$-$\mathcal{C}$-set.

Proposition 3.2.19 Let (X, m_x^1, m_x^2) be a biminimal structure space and M be a subset of X. If M is an $(i, j)m_x$-locally closed set, then it is also an $(i, j)m_x$-$\mathcal{C}$-set, where i, j = 1 or 2 and $i \neq j$.

Proof. Let M be an $(i, j)m_x$-locally closed set. Then there exist U and B such that $M = U \cap B$ where U is m_x^i-open in (X, m_x^i) and B is m_x^j-closed.
It follows that $m_x^jCl(m_x^jInt(B)) \subseteq m_x^jCl(B) = B$. Then B is an m_x^j-preclosed. Hence M is an $(i, j)m_x$-$\mathcal{C}$-set.

Proposition 3.2.20 Let A be a subset of a biminimal stucture space (X, m_x^1, m_x^2) and m_x^j has the property $\mathfrak{B}$. Then A is an $(i, j)m_x$-$\mathcal{C}$-set iff $A = U \cap m_x^jpcl(A)$ for some $U \in m_x^i$, where i, j = 1 or 2 and $i \neq j$.

Proof. ($\Longrightarrow$) Let A be $(i, j)m_x$-$\mathcal{C}$-set. Then there exist U and B such that $A = U \cap B$ where U is m_x^i-open and B is m_x^j-preclosed. From $A \subseteq B$, $m_x^jpcl(A) \subseteq m_x^jpcl(B)$ by Proposition 2.11, $m_x^jpcl(B) = B \cup m_x^jCl(m_x^jInt(B))$. As B is m_x^j-preclosed, $m_x^jCl(m_x^jInt(B)) \subseteq B$. Hence $m_x^jpcl(B) = B$. Thus $m_x^jpcl(A) \subseteq B$. Therefore $A = U \cap m_x^jpcl(A)$.

($\Longleftarrow$) Let $A = U \cap m_x^jpcl(A)$ for some $U \in m_x^i$. Since $m_x^jpcl(A)$ is an m_x^j-preclosed. Therefore, A is an $(i, j)m_x$-$\mathcal{C}$-set.

Proposition 3.2.21 Let A be a subset of a biminimal stucture space (X, m_x^1, m_x^2) and m_x^j has the property $\mathfrak{B}$. Then $A = U \cap m_x^jCl(m_x^jInt(A))$ for some $U \in m_x^i$ if and only if A is an m_x^j-semi-open and $(i, j)m_x$-$\mathcal{C}$-set, where i, j = 1 or 2 and $i \neq j$.

Proof. ($\Longrightarrow$) Let $A = U \cap m_x^jCl(m_x^jInt(A))$ for some $U \in m_x^i$. Then $A \subseteq m_x^jCl(m_x^jInt(A))$. Thus A is an m_x^j-semi-open. By Lemma 2.6, $m_x^jCl(m_x^jInt(A))$ is m_x^j-closed. Since $m_x^jInt(m_x^jCl(m_x^jInt(A))) \subseteq m_x^jCl(m_x^jInt(A))$, $m_x^jCl(m_x^jInt(m_x^jCl(m_x^jInt(A)))) \subseteq m_x^jCl(m_x^jInt(A))$. Hence $m_x^jCl(m_x^jInt(A))$ is m_x^j-preclosed. Then A is an $(i, j)m_x$-$\mathcal{C}$-set.

($\Longleftarrow$) Let A be an m_x^j-semi-open and $(i, j)m_x$-$\mathcal{C}$-set. By Proposition 3.2.20, $A = U \cap m_x^jpcl(A)$ for some $U \in m_x^i$. Since A is m_x^j-semi-open. Then $A \subseteq m_x^jCl(m_x^jInt(A))$. Since m_x^j has the property $\mathfrak{B}$ and by Proposition 2.11, $m_x^jpcl(A) = A \cup m_x^jCl(m_x^jInt(A))$. Thus $m_x^jpcl(A) = m_x^jCl(m_x^jInt(A))$. Hence $A = U \cap m_x^jCl(m_x^jInt(A))$

for some $U \in m_X^i$.

Theorem 3.2.22 Let A be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and m_X^j has the property $\mathfrak{B}$. If a subset M of X is an m_X^j-semi-open and $(i, j)m_X$-$\mathcal{C}$-set, then it is an $(i, j)m_X$-$\mathcal{A}$-set.

Proof. Let M be an m_X^j-semi-open and ($(i, j)m_X$ -$\mathcal{C}$-set. By Proposition 3.2.21, then $M = U \cap m_X^jCl(m_X^jInt(M))$, for some $U \in m_X^i$. Since $m_X^jCl(m_X^jInt(M))$ is m_X^j-regular closed. Therefore, M is an $(i, j)m_X$-$\mathcal{A}$-set.

Definition 3.2.23 Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces.

A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be

(1) (i, j)-semi-continuous if $f^{-1}(V) \in (i, j)\text{-}SO(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(1) (i, j)-$\mathcal{LC}$-continuous if $f^{-1}(V) \in (i, j)\text{-}\mathcal{LC}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(1) (i, j)-$\mathcal{A}$-continuous if $f^{-1}(V) \in (i, j)\text{-}\mathcal{A}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(1) (i, j)-$\mathcal{C}$-continuous if $f^{-1}(V) \in (i, j)\text{-}\mathcal{C}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

Proposition 3.2.24 Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ be a mapping. If f is (i, j)-$\mathcal{A}$-continuous then f is (i, j)-$\mathcal{LC}$-continuous.

Proof. Let f be (i, j)-$\mathcal{A}$-continuous and $V \in m_Y^i$. Then $f^{-1}(V) \in (i, j)\text{-}\mathcal{A}(X, m_X^1, m_X^2)$. By Lemma 3.2.7, we have $f^{-1}(V) \in (i, j)\text{-}\mathcal{LC}(X, m_X^1, m_X^2)$. Hence f is (i, j)-$\mathcal{LC}$-continuous.

Theorem 3.2.25 Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces and m_X^j has the property $\mathfrak{B}$. If a mapping $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is (i, j)-semi-continuous and (i, j)-$\mathcal{LC}$-continuous then f is (i, j)-$\mathcal{A}$-continuous.

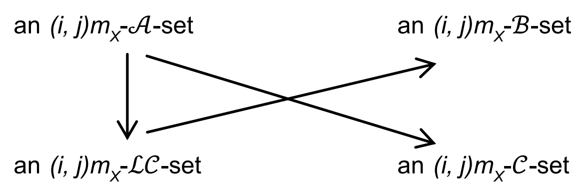
Proof. Let f be an (i, j)-semi-continuous and (i, j)-$\mathcal{LC}$ -continuous and $V \in m_Y^i$. Then $f^{-1}(V) \in (i, j)\text{-}SO(X, m_X^1, m_X^2)$ and $f^{-1}(V) \in (i, j)\text{-}\mathcal{LC}(X, m_X^1, m_X^2)$. By Proposition 3.2.8, thus $f^{-1}(V) \in (i, j)\text{-}\mathcal{A}(X, m_X^1, m_X^2)$. Therefore, f is (i, j)-$\mathcal{A}$-continuous.

Theorem 3.2.26 Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. If a mapping $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is (i, j)-semi-continuous and (i, j)-$\mathcal{C}$-continuous then f is (i, j)-$\mathcal{A}$-continuous.

Proof. Let f be an (i, j)-semi-continuous and (i, j)-$\mathcal{C}$-continuous and $V \in m_Y^i$. Then $f^{-1}(V) \in (i, j)\text{-}SO(X, m_X^1, m_X^2)$ and $f^{-1}(V) \in (i, j)\text{-}\mathcal{C}(X, m_X^1, m_X^2)$. By Theorem 3.2.22, thus $f^{-1}(V) \in (i, j)\text{-}\mathcal{A}(X, m_X^1, m_X^2)$. Therefore, f is (i, j)-$\mathcal{A}$-continuous.

Conclusion

In this paper, we introduced the concept of $\mathcal{A}$-sets in biminimal structure spaces. We also studied some properties of $\mathcal{A}$-sets and $\mathcal{A}$-continuous function on the space. The following implications hold for a biminimal structure spaces. These implications are not reversible.



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