# **ตัวกรองไฮเพอร์วิภัชนัยแบบอ่อนของพีชคณิตบีอีไฮเพอร์ Fuzzy Weak Hyper Filters of Hyper BE-Algebras**

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### **บทคัดย่อ**

ในบทความวิจัยนี้ได้แนะนำแนวคิดของตัวกรองไฮเพอร์วิภัชนัยแบบอ่อนในพีชคณิตบีอีไฮเพอร์ และได้ศึกษาสมบัติบางประการ ของตัวกรองไฮเพอร์วิภัชนัยแบบอ่อน จากนั้นได้แสดงว่าเซตของตัวกรองไฮเพอร์วิภัชนัยแบบอ่อนทั้งหมดของพีชคณิตบีอีไฮเพอร์ เป็นแลตทิซบริบูรณ์ที่มีการแจงแจง ยิ่งไปกว่านั้นได้จำ แนกลักษณะเฉพาะของพีชคณิตบีอีไฮเพอร์นอเทอร์เรียน และพีชคณิต บีอีไฮเพอร์อาร์ทิเนียน โดยใช้ตัวกรองไฮเพอร์วิภัชนัยแบบอ่อน

**คำ สำ คัญ**: ตัวกรองไฮเพอร์วิภัชนัย ตัวกรองไฮเพอร์วิภัชนัยแบบอ่อน พีชคณิตบีอี พีชคณิตบีอีไฮเพอร์

### **Abstract**

The aim of this work is to introduce the notion of fuzzy weak hyper filters in hyper BE-algebras and investigate some of their properties. This research shows that the set of all fuzzy weak hyper filters of hyper BE-algebras is a distributive complete lattice. Also, the concepts of Noetherian hyper BE-algebras and Artinian hyper BE-algebras are characterized by their fuzzy weak hyper filters.

**Keywords**: fuzzy hyper filter, fuzzy weak hyper filter, BE-algebra, hyper BE-algebra

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### **Introduction**

The fuzzy set was introduced by  $Z$ adeh<sup>1</sup> as a function from a nonempty set  $X$  to the unit interval  $[0,1]$ . Later, many researchers have discussed the generalizations of the concepts of fuzzy sets with applications in computing, logic and many ramifications of pure and applied mathematics. Kim and  $Kim<sup>2</sup> introduced the notion of  $\frac{1}{2}$$ BE-algebras, as a generalization of BCK-algebras $^3$  and BCI-algebras<sup>4</sup>. In 2010, the concept of fuzzy ideals in BE-algebras was introduced and some of its properties were investigated by Song, Jun and Lee<sup>5</sup>. Then, Dymek and Walendziak<sup>6</sup> studied and characterized the concept of fuzzy filters in BE-algebras. some types of hyper filters on hyper BE-algebras.

The hyperstructure theory was introduced by Marty<sup>7</sup> in 1934 as a generalization of ordinary algebraic structures. Radfar, Rezaei and Borumand Saeid<sup>8</sup> applied the hyper theory to introduce the notion of hyper BE-algebras, as a generalization of BE-algebras. In 2015, Cheng and Xin<sup>9</sup> investigated some types of hyper filters on hyper BE-algebras.

In this work, the concept of fuzzy weak hyper filters of hyper BE-algebras is introduced, and its properties are considered. Finally, the concepts of Noetherian **Preliminaries** hyper BE-algebras and Artinian hyper BE-algebras are myper DE algebras and ∧tunian hyper DE algebras.<br>Characterized by their fuzzy weak hyper filters.

## **Preliminaries**

Let  $X$  be a nonempty set. The mapping  $\circ$ ,  $X$   $x$  $X \rightarrow P^*(X)$ , where  $P^*(X)$  denotes the set of all nonempty subsets of *H*, is called a *hyperoperation*<sup>10-13</sup> on *H*. The hyperstructure *(H,*O*)* is called a *hypergroupoid.* Let *A* and *B* be any two nonempty subsets of  $H$  and  $x \in H$ . Then, we denote no don *het X* be a nonempty set. The mapping on . The mapping of  $\mathbb{R}^n$ 

$$
A \circ B = \bigcup_{a \in A, b \in B} a \circ b,
$$
  

$$
A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.
$$

Let *H* be a nonempty set and  $\circ$ :  $X \times X \rightarrow P^*(X)$ be a hyperoperation. Then  $(H, o, I)$  is called a *hyper* BE-algebra<sup>8</sup> if it satisfies the following axioms: *h*e a hyperoperation. Then  $(H, 0, I)$  is called a *hy*<br>PE-clocked<sup>8</sup> if it satisfies the following axioms:

(i) 
$$
x < 1
$$
 and  $x < x$ ;  
(ii)  $x \circ (y \circ z) = y \circ (x \circ z)$ ;

(iii) 
$$
x \in I
$$
 o x;  
(iv)  $I < x$  implies  $x = I$ ;

for all  $x, y, z \in H$ , where the relation " $\lt$ " is defined by  $x \leq y$  if and only if  $I \in x \circ y$ .

**Example 2.1**<sup>8</sup> Define the hyperoperation "o" on  $\mathbb R$  as follows:

$$
x \circ y = \begin{cases} \{y\} & \text{if } x = 1; \\ \mathbb{R} & \text{otherwise.} \end{cases}
$$

Then, (ℝ,∘, 1) is a hyper BE-algebra. Then, *( ,O,1)* is a hyper BE-algebra. **Example 2.28** Let  $\mathcal{L}$ 

**Example 2.2**<sup>8</sup> Let  $X = \{1, a, b\}$ . Define the hyperoperation "<sup>o"</sup> on as follows:

$\overline{O}$		a	
	{1}	${a}$	{b}
a	$\{I,a\}$	$\{1,a,b\}$	$\{1,a\}$
h	$\{1,a,b\}$	${a}$	$\{1,a,b\}$

Then,  $(H, o, I)$  is a hyper BE-algebra.

Let  $F$  be a nonempty subset of a hyper  $BE$ -algebra *H* and *I* ∈ *F*. Then *F* is called:

(i) a *weak hyper filter*<sup>8</sup> of *H* if  $x \circ y \subseteq F$  and  $x \in F$ , (ii) a *hyper* filter<br>8 of is ≈ and if is ≈ and if it is <u>in</u> it is  $\frac{1}{2}$ then  $y \in F$ , for all  $x, y \in F$ ;

 $y \in F$ , where  $x \circ y \approx F$  means that  $x \circ y \cap F \neq \emptyset$ , for all *x,y*  $\in H$ . (ii) a *hyper filter*<sup>8</sup> of *H* if  $x \circ y \approx F$  and  $x \in F$ , then ∈ *H*.

Note that every hyper filter of a hyper BE-Note that every hyper filter of a hyper BE-algebra  $H$  is a weak hyper filter of  $H$ , but the converse is not true in general<sup>8</sup>. In this paper, we will focus on weak hyper filters of hyper BE-algebras.

**Lemma 2.3** If  $\{F_i: i \in I\}$  is a chain of a family of weak hyper filters of a hyper BE-algebra  $H$ , then  $\mathop{\rm {}U\!}\nolimits_{i\in I}F_i$  is also a weak hyper filter of *H*. *i*∈*I*

**Proof.** Let  $\bigcup_{i \in I} F_i$ . Clearly,  $I \in F$ . Let  $x, y \in H$  such that  $x \, o \, y \subseteq F$  and  $x \in F$ . Then  $x \, o \, y \subseteq F_i$  and  $x \in F_i$  for some *i*,*j*∈*I*. Assume that  $F_i ⊆ F_j$ : It follows that *x o y* ⊆  $F_j$  and *x*∈*F<sub>j</sub>*. Since *F<sub>j</sub>* is a weak hyper filter of *H*, we have  $y \in$  $F_i \subseteq F$ . Hence, *F* is a weak hyper filter of *H*.

A *fuzzy set*<sup>1</sup> of a nonempty set  $X$  is a mapping µ: *X* → [*0,1*]. Then, the set U(µ*;*α)={x∈X:µ(x)≥α} is called a *level subset* of  $\mu$ *.* where  $\alpha \in [0,1]$ *.* Let  $\mu$  and  $\nu$  be any two fuzzy sets of a nonempty set *X*. Then  $\mu \leq v$ , means

that  $\mu$ (x)≤v(x), for all *x*∈*X*. In addition, the intersection for some  $\mu$ and the union of  $\mu$  and  $\nu$ , denoted by  $\mu \cap \nu$  and  $\mu \cup \nu$ , respectively, are defined by letting  $x \in X$ ,  $(\mu \cap \nu)$ (x)=min{ $\mu$ (x), v(x)} and ( $\mu$ ∪*v*)(x)=max{ $\mu$ (x), v(x)}.  $(x)=\min\{\mu(x), v(x)\}\$ and  $(\mu\cup v)(x)=\max\{\mu(x), v(x)\}\$ . and  $\alpha<\mu(\lambda)$ d by  $\mu \cap \nu$  and  $\mu \cup \nu$ y letting  $x \in X$ ,  $(\mu \cap \nu)$  $\mu$ <sup>1</sup>  $\nu$  and  $\mu$   $\sigma$  $\int u(x) v(x)$ �∈�∘�

#### **Results** for all , ∈ .

In this section, we introduce the notion of fuzzy weak hyper filters of hyper BE-algebras, and we investigate some fundamental properties of fuzzy weak hyper filters in hyper BE-algebras.

**Definition 3.1** A fuzzy set µ of a hyper BE-algebra *H* is called a *fuzzy weak hyper filter* of *H* if it satisfies the following conditions:

$$
(i) \mu(1) \ge \mu(x);
$$

(ii) 
$$
\mu(x) \ge \min\{\inf_{z \in yx} \mu(z), \mu(y)\};
$$

for all  $x, y \in H$ .

**Example 3.2** Let  $H = \{1, a, b\}$  be a set with a hyperoperartion "o" on defined as follows:

$\overline{O}$		a		
	{1}	${a,b}$	${b}$	
a	{1}	$\{1,a\}$	$\{1,b\}$	
h	{1}	$\{1,a,b\}$	{]}	

Then, is a hyper  $BE$ -algebra $^8$ . We define a fuzzy set  $\mu$  of *H* by  $\mu$ (a)  $\leq \mu$ (b)  $\leq \mu$ (1). By routine computations, we have that  $\mu$  is a fuzzy weak hyper filter of  $H$ .

**Theorem 3.3** Let be a fuzzy set of a hyper BEalgebra  $H$ . Then  $\mu$  is a fuzzy weak hyper filter of  $H$  if and only if its nonempty level subset  $U(\mu;\alpha) = \{x \in H : \mu(x) \ge \alpha\}$ is a weak hyper filter of for all  $\alpha \in [0,1]$ .

**Proof.** Assume that  $\mu$  is a fuzzy weak hyper filter of *H*. Let  $\alpha \in [0,1]$  such that  $U(\mu;\alpha) \neq \emptyset$ . Then there exists  $x_0 \in U(\mu;\alpha)$  such that  $\mu(x_0) \geq \alpha$ . Since  $\mu(I) \geq \mu(x_0)$ ,  $I \in U(\mu;\alpha)$ . Let  $x, y \in H$  such that  $x \circ y \subseteq U(\mu; \alpha)$  and  $x \in U(\mu; \alpha)$ . Then  $μ(z) ≥ α$ , for all  $z ∈ x o y$ . Thus,  $μ(y) ≥ min{inf_{z ∈ ω} μ(z), μ(x)} ≥ α$ ,  $\mu(z) \ge \alpha$ , for all  $z \in x$  *oy*. Filus,  $\mu(y) \ge \min\{\lim_{z \in x} \mu(z), \mu(x)\} \ge \alpha$ , that is,  $y \in U(\mu;\alpha)$ . Hence,  $U(\mu;\alpha)$  is a weak hyper filter of *H*.

Conversely, suppose that  $\mu(I) \geq \mu(x_0) = \beta$  for some  $\mathcal{X}_0 \in H$  and  $\beta \in [0,1]$  . Then  $U(\mu;\beta) \neq \emptyset$ , and so  $U(\mu;\beta)$  is a weak hyper filter of *H*. It follows that  $I \in U(\mu;\beta)$ , which implies that µ(*1*)≥β. This is a contradiction. Thus, µ(*1*)≥µ(*x*), for all  $x \in H$ . Suppose that  $\mu(a) < \min\{\inf_{x \in A} \mu(z), \mu(b)\}$ *z*∈*boa*

for some *a,b*∈*H*. Letting  $\mu(b)$ . . ome  $a, b \in H$ . Letting  $\alpha = \frac{1}{2} \Big( \mu(a) + \min \Big\{ \inf_{z \in b \circ a} \mu(z), \Big\}$  $\{\}.$ 

 $\vee$ , () weak hyper filter of *H*, we have  $a \in U(\mu;\alpha)$ , that is,  $\mu(a) \ge \alpha$ . We have  $\mu(a) < a < \min\{\inf_{z \in ba} \mu(z), \mu(b)\} \le \inf_{z \in ba} \mu(z)$ and  $\alpha<\!\!\mu(b)$ . Then  $b$ o $a$   $\subseteq$   $U(\mu;\!\alpha)$  and  $b$   $\in$   $U(\mu;\!\alpha)$ . Since is a This is a contradiction. We obtain that  $\mu(a) \geq \min \{ \inf_{z \in ba} \mu(z), \}$  $\mu(b)$ } for all *a,b*∈*H*. Therefore, μ is a fuzzy weak hyper filter of  $H$ . *z*∈*boa z*∈*boa* hyper litter of  $H$ , we have  $a \in U(\mu, a)$ , that is,  $\mu(a) \ge a$ .<br>is a contradiction Me obtain that  $\nu(a)$  min  $\int \nu(a) \cdot$ 

> **Corollary 3.4** If  $\mu$  is a fuzzy weak hyper filter of a hyper BE-algebra *H*, then the set  $H_a = \{x \in H : \mu(x) \ge \mu(a)\}$ is a weak hyper filter of *H* for all *a*∈*H*.

> **Corollary 3.5** If  $\mu$  is a fuzzy weak hyper filter of a hyper BE-algebra *H*, then the set  $H_{\mu} = {x \in H : \mu(x) = \mu(I)}$ is a weak hyper filter of  $H$ .  $\frac{1}{2}$  is a weak hyper-filter of  $\frac{1}{2}$  for an  $\frac{1}{2}$ .

> **Theorem 3.6** Let  $F_{\iota} \subset F_{\iota} \subset \cdots F_{\iota} \subset \cdots$  be a strictly ascending chain of weak hyper filters of a hyper BE-algebra *H* and  $\{t_n\}$  be a strictly decreasing sequence in  $[0,1]$ . Let  $\mu$  be a fuzzy set of *H*, defined by  $\mu(x)$ =

$$
\begin{cases}\n0 & \text{if } x \notin F_n \text{ for each } n \in \mathbb{N}; \\
t_n & \text{if } x \in F_n - F_{n-1} \text{ for } n = 1, 2, \dots;\n\end{cases}
$$

for all  $x{\in}H$ , where  $F_{\overline{\theta}}\!\!=\!\!\varnothing$ . Then  $\mu$  is a fuzzy weak hyper filter of  $H$ .

**Proof.** Let  $F = \underset{n \in \mathbb{N}}{U} F_n$ . By Lemma 2.3, F is a weak hyper filter of *H*. Then  $\mu(I)=t_1\geq \mu(x)$ , for all  $x\in H$ . Let  $x,y\in H$ . Thus, we can divide to be two cases, as follows.

Case 1:  $x \notin F$ . Then  $y \circ x \notin F$  or  $y \notin F$ . There exists  $a \in y \circ x$  such that  $x \notin F$ . Thus,  $\mu(a)=0$  or  $\mu(y)=0$ . Hence,  $\min\{\inf_{z \in \mathbb{N}} \mu(z), \mu(y)\}.$ *z*∈*yox*

Case 2:  $x \in F_n - F_{n-1}$  for some  $n = 1, 2, ...$  Then *y*  $\circ$  *x* $\not\subseteq$ *F<sub>n-1</sub>* or *y*∉*F*. Thus, there exists *a*∈*y*  $\circ$  *x* such that  $a \notin F_{n-1}$ . We obtain that,  $\inf_{z \in \mathcal{X}} \mu(z) \le t_n$  or  $\mu(y) \le t_n$ . Therefore,  $\min\left\{\inf_{z\in y\circ x}\mu(z),\mu(y)\right\}\leq t_n=\mu(x)$ . Consequently,  $(2e^{j\omega} \mu)$  is a fuzzy weak hyper filter of *H*.

The *cartesian product*<sup>14</sup> of  $\mu$  and  $\nu$  is defined by  $(\mu x \nu)$ Let  $\mu$  and  $\nu$  be fuzzy sets of a nonempty set X.  $(x, y) = \min\{\inf_{z \in y} \mu(z), \mu(b), \text{ for all } x, y \in X.\}$ *z*∈*yox*

 $\sum_{z \in \mathcal{X}} P(z)$ ,  $P(z)$ , is defined in  $\sum_{z \in \mathcal{Y}} P(z)$  $\mu$  and *v* are fuzzy weak hyper filters of *H*, then  $\mu x v$  is a fuzzy weak hyper filter of  $H x H$ . **Theorem 3.7** Let *H* be a hyper BE-algebra. If

**Proof.** Assume that  $\mu$  and  $\nu$  are fuzzy weak hyper filters of  $H$ . Let  $(x,y) \in H x H$ . Then

 $(\mu \times \nu)(1, 1) = \min{\mu(1), \nu(1)} \ge \min{\mu(x), \nu(y)}$   $\beta \mu(x) +$  $= (\mu \times \nu)(x, y)$ . Now, let  $(x_1, y_2), (x_2, y_2) \in H \times H$ . Then �∈�∘� �,�  $(\mu, \nu)$   $(x_i, y_j)$  $= (\mu \wedge \nu)(\lambda, \gamma)$ . No  $\qquad \qquad (1)$ 

$$
= \min{\mu(x_1), \nu(y_1)}
$$
  
\n
$$
\geq \min{\min{\prod_{z_1 \in x_2 \circ x_1}} \mu(z_1), \mu(x_2)},
$$
  
\n
$$
\min{\min{\prod_{z_2 \in y_2 \circ y_1}} \nu(z_2), \nu(y_2)}
$$
  
\n
$$
\geq \min{\prod_{z_1 \in x_2 \circ x_1} \{\min{\mu(z_1), \nu(z_2)}\},
$$
  
\n
$$
\frac{z_2 \in y_2 \circ y_1}{z_2 \in y_2 \circ y_1}
$$
  
\n
$$
\min{\mu(x_2), \nu(y_2)}
$$
  
\n
$$
\geq \min{\sum_{(z_1, z_2) \in (x_2, y_2) \circ (x_1, y_1)} (\mu \times \nu) (z_1, z_2)}
$$
  
\n
$$
(\mu \times \nu)(x_2, y_2)
$$
.

Therefore,  $\mu x y$  is a fuzzy weak hyper filter of  $H x H$ .<br>Therefore,  $\mu x y$  is a fuzzy weak hyper filter of  $H x H$ .

Let be a fuzzy set of a nonempty set *X*,  $\alpha \in [0,1-\alpha]$  of with res  $\sup_{x \in X} \mu(x)$  and  $\beta \in [0,1]$ . Then: Let be a fuzzy set of a nonempty set X,  $\alpha \in [0,1-\alpha]$  of with respectively  $f$  and the state of  $f$  . Let  $h$ e a function  $f$ *x*∈*X*

(i) the mapping  $\mu^T a : X \to [0,1]$  is called a  $\mu^T a : X \to [0,1]$  is called a fuzzy translation<sup>15</sup> of  $\mu$  if  $\mu^T_a(x)=\mu(x)+a$ , for all  $x \in X$ ;<br>
(i)  $\mu$  is a fuzzy weak hyper-(i) the mapping  $\mu^T_{a}: X \rightarrow [0,1]$  is calle fuzzy translation<sup>15</sup> of  $\mu$  if  $\mu^T_a(x)=\mu(x)+a$ , for all  $x \in X$ ; (i) the mapping  $\mu^T_a$ :  $X \rightarrow [0,1]$  is called a  $(i)$  the

(ii) the mapping  $\mu_{\beta}^{M}: X \rightarrow [0,1]$  is called a fuzzy multiplication<sup>15</sup> of  $\mu$  if  $\mu^M_{\beta}(x)=\beta\mu(x)$ , for all  $x \in X$ ; (iii)  $\mu^M_{\beta}$  is a fuz (ii) the mapping  $\mu^{M}_{\beta}$ :  $X \rightarrow [0,1]$  is called a (ii)  $\mu^{M}_{\alpha}$  is a fuzzy weak hy<br>(iii)  $\mu^{M}_{\alpha}$  is a fuzzy weak hy *fuzzy multiplication*<sup>15</sup> of  $\mu$  if  $\mu^M{}_\beta(x) = \beta \mu(x)$ , for all  $x \in X$ ; (ii) the mapping  $\mu^{M}{}_{\beta}$ :  $X \rightarrow [0,1]$  is called a

(iii) the mapping  $\mu_{\beta,\alpha}^{MT}: X \to [0,1]$  is called a<br>filters of a hyper BEfuzzy magnified translation<sup>16</sup> of  $\mu$  if  $\mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$ , weak hy **Theorem 3.8** Let be a hyper BE-algebra, be a for all  $x \in X$ . fuzzy magnified translation<sup>16</sup> of  $\mu$  if  $\mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$ , filte<br>for all  $\kappa \in Y$ **Theorem 3.8** Let  $A \in \Lambda$ ,  $wcan$  iiy all est en  $\kappa \in X$ .  $f(x) = \int_0^x f(x) dx + \int_0^x f(x) dx$  (iii) the mapping  $\mu_{B,\alpha}^{MT}: X \to [0,1]$  is called a fuzzy magnified translation<sup>16</sup> of  $\mu$  if  $\mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$ ,  $iX$ . for all  $x \in X$ .  $\lambda \in \Lambda$ .

 $\ddot{\phantom{a}}$  ,  $\ddot{\phantom{a}}$ **Theorem 3.8** a fuzzy set of  $H \alpha \in [0, 1 - \sup \mu(x)]$  and  $\beta \in$ a fuzzy set of *H*,  $\alpha \in [0,1]$  sup  $\mu(x)$  and  $\mu \in [0,1]$ .<br>Suppose that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy magnified translation of  $\mu$ , With respect to *α* and β. Then μ is a fuzzy weak hyper  $\mu(\mu \cap \nu)(1) = \min{\mu(1), \nu(1)}$ <br>  $\geq \min{\mu(x), \nu(x)} =$  (*μ* with respect to  $\alpha$  and  $p$ . Then  $\mu$  is a fuzzy weak is  $\int_{r}^{t} p_{y} d\theta$  , with respect to  $\theta$  is  $\int_{r}^{t} p_{y} d\theta$  . Then  $\int_{r}^{t} p_{y} d\theta$  , with  $\$ **Pro**<br>**Theorem 3.8** Let  $H$  be a hyper BE-algebra,  $\mu$  be<br>filters of a by translation of  $\mu$ ,  $(\mu \cap \nu)(1) = m$  $\frac{3}{2}$  fuzzy weak hyper filter of  $\geq$ a fuzzy weak hyper filter of  $\alpha$ a fuzzy set of *H*,  $\alpha \in [0,1-\sup_{x \in H} \mu(x)]$  and  $\beta \in [0,1-\min_{x \in H} \mu(x)]$ Suppose that  $\mu_{\beta,\alpha}$  is a fuzzy magnified translation of then of  $H$  if and only if  $u^{MT}$  is a fuzzy weak byper fil  $\mathsf{of} \ H.$ and  $\beta \in [0,1]$ . Suppose that fuzzy set of H,  $\alpha \in [0,1-\sup_{x \in H} \mu(x)]$ Suppose that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy magnified translation of  $\mu$ , and only if  $\mu_{\beta,\alpha}$  is a fuzzy weak<br>of H  $\text{def}\$  and  $\text{and}\$ the state of a fuzzy set of H,  $\alpha \in [0,1-\sup_{x \in H} \mu(x)]$  and  $\beta \in [0,1-\sup_{x \in H} \mu(x)]$ of  $H$ . of *H*. with respect to  $\alpha$  and  $\beta$ . Then  $\mu$  is a fuzzy weak hyper  $\mu \cap \nu$  = min{ $\mu$ (1),  $\nu$ (1)}<br> $\mu \cap \nu$  = min{ $\mu$ (1),  $\nu$ (1)} = min{ $\mu$ (x),  $\nu$ (x)} = **Proper filter and** a fuzzy set of H,  $\alpha \in [0,1-\sup \mu(x)]$ Suppose that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy magnified to ,<br>, f a fuzzy set of H,  $\alpha \in [0,1-\sup_{x\in H}\mu(x)]$  and  $\beta \in [0,1]$ .<br>Suppose that  $\mu_{\alpha}^{MT}$  is a fuzzy magnified translation of u.  $\mathcal{L} = \mathcal{L} \mathcal{L}$  $\Gamma$ Suppose that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy magnified translation of  $\mu$ ,  $(\mu \cap \nu)(1) = \min{\mu(1), \nu(1)}$  $\mathsf{p}$  of  $H$ . respect to  $\alpha$  and  $\beta$ . Then  $\mu$  is a fuzzy weak hyper  $(\mu \cap \nu)$ **Provide that and and**  $\theta$ a fuzzy set of  $H, G$ **Theorem 3.8** Let H be a l e a hyper BE-algebra,  $\mu$  be  $x \in H$ <br>see that  $u^{MT}$  is a fuzzy meanifed translation of  $u$ of *H* if and only if  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy w **Proof.** Assume that is a fuzzy weak hyper filter of **Proof.** Theorem 3.8 Let *H* be a hyper BE-algebra,  $\mu$  be  $x \in \widetilde{H}$   $x \in \widetilde{H}$   $x \in \widetilde{H}$   $x \in \widetilde{H}$  and  $p \in [0,1]$ .  $\ddot{\phantom{0}}$ **Problem 5.6** Let  $H$  be a hyper BL-aigebia,  $\mu$  be<br>  $\mu$  ext of  $H \propto F \left[ 0.1 - \sin \mu(x) \right]$  and  $R = [0.11]$  $\mu_{B,\alpha}^{MT}$  is a fuzzy magnified translation of μ,

**Proof.** Assume that  $\mu$  is a fuzzy weak hyper filter  $(\mu \cap \nu)(x) = m$ **Proof.** Assume that  $\mu$  is a fuzzy weak hyper filter<br>
of *H*. Let  $a \in H$ . Since  $\mu(I) \ge \mu(a)$ , we have  $\mu_{\beta,\alpha}^{MT}(1) = \qquad \qquad \ge \min\{\min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\},\}$  $\beta\mu(1)+\alpha\geq\beta\mu(a)+\alpha=\mu^{MT}_{\beta,\alpha}(a)$ , for all  $a{\in}H$ . Let  $\beta\mu(1) + \alpha \geq \beta\mu(a) + \alpha = \mu_{\beta,\alpha}^{MT}(a)$ , for all  $a \in H$ . Let  $\min\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}$  $\lambda, \lambda \in H$ . Then<br> $\overline{MT}(x) = 0$   $\overline{Q}(x) + \overline{Q}(x)$ **Proof.** Assume that  $\mu$  is a fuzzy weak hyper filter  $\mu(\mu \cap \nu)(x) = \min{\mu(x), \nu(x)}$ <br>  $\mu^T(a) \ge \min{\min{\min{\min{\mu(z), \mu(y)}}}$  $\mathcal{L}$ ,  $\mathcal{L}$  +  $\mathcal{L}$  =  $\mathcal{L}$  =  $\mathcal{L}$  +  $\mathcal{L}$  =  $\mathcal{L}$  +  $\mathcal{L}$  =  $\mathcal{L}$  +  $\mathcal{L}$  =  $\mathcal{L}$  +  $\$ of *H*. Let  $a \in H$ . Since  $\mu(I) \ge \mu(a)$ , we have  $\mu_{\beta,\alpha}^{MT}(1) = \qquad \qquad \ge \min\{\min\{\inf_{z \in \mathcal{Y}^{\circ}x} \mu(z), \mu(z)\}$  $\beta\mu(1) + \alpha \geq \beta\mu(a) + \alpha = \mu_{\beta,\alpha}^{MT}(a)$ , for all  $a \in H$ . Let min  $\mu(\alpha)$  is  $\mu_{\beta,\alpha}(\alpha)$ **Proof.** Assume that  $\mu$  is a fuzzy weak hyper filter  $\mu(x) = \min\{\mu(x), \nu(x)\}$ <br>  $\geq \min\{\min\{\inf_{x \in \mathcal{X}} \mu(x)\}$  $f \circ \epsilon \sim \mu_{\beta,\alpha}(1)$  $\beta \mu(1) + \alpha \geq \beta \mu(a) + \alpha = \mu_{\beta,\alpha}^{\alpha}(a)$ , for an  $a \in H$ . Let  $x \vee \in H$ . Then **Proof.** Assume that  $\mu$  is a fuzzy weak hyper lifter<br>
of *H*. Let  $a \in H$ . Since  $\mu(I) \ge \mu(a)$ , we have  $\mu_{\beta,\alpha}^{MT}(1) =$   $\ge \min\{$  $\overline{M}$  $\alpha > \beta u(a) + \alpha = u_n^M$ , we have  $r \beta u^{(2)}$ .<br>  $\beta u(a) + \alpha = u_n^M$ , for all  $a \in H$ . Let  $\ddot{\theta}$ ,  $\ddot{\theta}$ ,  $\ddot{\theta}$ �,� �� () = () +  $(\theta) + \alpha \geq \beta \mu(a) + \alpha = \mu_{B,\alpha}^{MT}(a)$ , for all  $a \in H$ . Let

$$
\mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha
$$
  
\n
$$
\geq \beta \min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\} + \alpha
$$
  
\n
$$
= \min\{\inf_{z \in y \circ x} (\beta \mu(z) + \alpha), \beta \mu(y) + \alpha\}
$$
  
\n
$$
= \min\{\inf_{z \in y \circ x} \mu_{\beta,\alpha}^{MT}(z), \mu_{\beta,\alpha}^{MT}(y)\}.
$$

 $\cdots$  music,  $\mu_{\beta,\alpha}$  is an  $\sum_{z \in y \circ x^+} P, \alpha \leftrightarrow P, \alpha \leftrightarrow P,$ Hence,  $\mu_{R,\alpha}^{MT}$  is a fuzzy weak hyper filter of  $H$ . Hence,  $\mu_{\beta,\alpha}$  is a luzzy weak hyp  $= \min_{z \in y \circ x} \mu_{\beta,\alpha}(z), \mu_{\beta,\alpha}(z)$ <br>Hence,  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy weak hyper filter of Hence, �,� Fields,  $\mu_{\beta,\alpha}$  is a fuzzy weak hyper-Hence,  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy weak hyper filter of H.  $Hence$   $\mu^{MT}$  is a fuzzy weak ce,  $\,\mu_{{\beta},\alpha}^{MT}\,$  is a fuzzy weak hyper filter of  $H$ .

 $\epsilon$  , and  $\epsilon$  ,  $\epsilon$  and  $\epsilon$   $= 50.105655$ , assume that  $r^2 \beta_i \alpha_i$ .<br>hyper filter of H. Let  $x, y \in H$ . Consider  $\mu_{\beta,\alpha}^{MT}(1) \ge \mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$  and  $\mu_{\beta,\alpha}$  is a luzzy weak **Droof** by per filter of H. Let  $x, y \in H$ . Consider  $\beta \mu(1) + \alpha = M^T (1)$ . myper liner of *H*. Let  $x, y \in H$ . Consider  $\beta \mu(1) + \alpha$ <br>  $\mu_{\beta,\alpha}^{MT}(1) \ge \mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$  and Conversely, assume that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy weak<br>Conversely, assume that  $\mu_{\beta,\alpha}^{MT}$  is a fuzzy weak hyper filter of H. Let  $x, y \in H$ . Consider  $\beta$  $F_{\beta,\alpha}(\gamma) = F_{\beta,\alpha}(\gamma)$  p<sub>r</sub>( $\alpha$ ) is and Conversely, assume that  $\mu_{\beta,\alpha}^{\beta,\alpha}$ . myper filter of H. Let  $x, y \in H$ . Considering  $\mu_{R,\alpha}^{MT}(1) \geq \mu_{R,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$  and  $\mu_{\beta,\alpha}^{MT}(1) \geq \mu_{\beta,\alpha}^{MT}(x) = \beta \mu(x) + \alpha$  and hyper filter of *H*. Let *x,y*∈*H*. Consider  $\beta \mu(1) + \alpha =$  **Proof.** Assume that  $\mu$  and  $\mu$  a  $\lim_{M \to \infty} E = \lim_{n \to \infty} E_n$  Consider  $\beta \mu(1) +$  $\mu_{\beta}(\mu) \leq \mu_{\beta,\alpha}(\mu) - \mu(\mu) + \alpha$  and �∈�∘� �,�  $= r \cdot p, u \cdot y \cdot p \cdot r \cdot y \cdot w \cdot y$ 

$$
\beta\mu(x) + \alpha = \mu_{\beta,\alpha}^{MT}(x)
$$
  
\n
$$
\geq \min\{\inf_{z \in y \circ x} \mu_{\beta,\alpha}^{MT}(z), \mu_{\beta,\alpha}^{MT}(y)\}
$$
  
\n
$$
= \min\{\inf_{z \in y \circ x} (\beta\mu(z) + \alpha), \beta\mu(y) + \alpha\}
$$
  
\n
$$
= \min\{\beta(\inf_{z \in y \circ x} \mu(z)) + \alpha, \beta\mu(y) + \alpha\}
$$
  
\n
$$
= \beta \min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\} + \alpha.
$$

Since  $\beta > 0$  and  $\alpha \ge 0$ , we have  $\mu(x) \ge \min\left\{\inf_{z \in y \circ x} g\right\}$ Fair  $\lambda, \gamma \in \mathbb{H}$ . Hence,  $\mu$  is a  $\{\},$  cince  $\beta > 0$  and  $\alpha > 0$ , we  $\mu(z), \mu(y)$  and  $\mu(l) \ge \mu(x)$  $v)$  ( $z_1$ ,  $z_2$ ), fuzzy weak hyper filter of *H*.  $\omega e$   $\mu(x) > \min \frac{1}{x}$  inf  $\alpha$ ll *x*,γ∈*H*. Hence, μ is a and  $\mu$ (*I*)≥ $\mu$ (*x*), for all *x*,*y*∈*H*. Hence,  $\mu$  is a Since  $\beta > 0$  and  $\alpha > 0$  we have  $\mu(x) > \min \{ \inf$  $\Big\{z\Big(\bigvee\limits_{i=1}^N\mu(y)\Big\}$  and  $\mu(I) \geq \mu(x)$ , for all  $x,y\in H$ . Hence,  $\mu$  is a

ر در در در **Corollary 3.9** Let *H* be a hyper BE-algebra, μ be a fuzzy set of  $H$ ,  $\alpha \in [0,1-\sup_{x \in X} \mu(x)]$ , and  $\beta \in [0,1]$ . Suppose per filter of  $H x H$ .<br>that  $\mu^T a$  is a fuzzy translation and is a fuzzy multiplication pect to and, respectively. Then the following  $\mathbf g$  is a fuzzy translation and  $\mathbf g$  $\ddot{\mathbf{z}}$ � is a fuzzy translation and conditions are equivalent: Let be a fuzzy set of a nonempty set , (0,1]. Suppose that � fual  $\mu_a$  is a fuzzy diaristation and is a fuzzy maniphication<br>of with respect to and, respectively. Then the following  $\mathcal{A}$ �∈� fuzzy set of  $H$ ,  $\alpha \in [0,1-\sup_{x \in \mathbb{R}} \mu(x)]$ , and  $\beta \in [0,1]$ . Suppose onditions are equ

a  $(i)$   $\mu$  is a fuzzy weak hyper filter of  $H$ ;  $H$ ; )  $\mu$  is a fuzzy weak hyper filter of  $H$  ;  $\mathsf{r}$  of  $H$   $\cdot$ 

 $\lim_{n \to \infty}$  (ii)  $\mu_{a}^{T}$  is a fuzzy weak hyper filter of *H*;<br>is called a and  $r$  or  $H$ ; or all  $x \in X$ ;<br>(ii)  $\mu^T{}_a$  is a fuzzy weak hyper filter of  $H$ ; filter of  $H^+$ 

all  $x \in X$ ; (iii)  $\mu_{\beta}^M$  is a fuzzy weak hyper filter of H. filter of  $H$  $\mathcal{X}$ ; (iii)  $\mu^M{}_\beta$  is a fuzzy weak hyper filter of  $H$ .

**Theorem 3.10** If  $\mu$  and  $\nu$  are fuzzy weak hyper filters of a hyper BE-algebra *H*, then  $\mu \cap v$  is a fuzzy weak hyper filter of  $H$ .  $\mu$  and  $\nu$  are following weak tryper  $\frac{1}{2}$ for all  $x \in X$ ;<br> **Theorem 3.10** If  $\mu$  and  $\nu$  are fuzzy weak hyper called a<br>
filters of a hyper BE-algebra H, then  $\mu \cap v$  is a filters of a hyper BE-algebra H, then  $\mu \cap \nu$  is a  $\Box$  is called a<br> $\Box$  is called a  $\beta \mu(x) + \alpha$ , weak hyper filter of  $H$ .  $(i)$  is a fuzzy weak hyper filter of  $(i)$  $\mathcal{L}$  $\alpha$ , weak hyper  $\beta$  angusta  $\alpha$ ; [0,1] is called a<br>
filters of a hyper BE-algebra H, then  $\mu \cap v$  is a function of  $\mu$  is a function of  $\mu$ .  $\frac{1}{2}$  is a fuzzy value of  $\frac{1}{2}$ .  $u$  is a fuzzy weak hyper filter  $u$  $\alpha$ , weak hyper filter of H. fers of a hyper BE-algebra  $H$ , then  $\mu \cap v$  is a fuzzy reak hyper liiter of  $H$ .  $\mathbf{F}$  or  $\mathbf{H}$ .

**Proof.** Assume that  $\mu$  and  $\nu$  are fuzzy weak hyper filters of a hyper BE-algebra *H*. Let *x*,*y*∈*H*. Then are equivalent: filters of a hyper BE-algebra  $H$ . Let  $x,y \in H$ . Th  $\frac{1}{2}$  is a fuzzy weak hypers filter of . are equivalent<br>Externa equivalent:  $\mathcal{L}$  is a fuzzy weak hypers filter of  $\mathcal{L}$ E-algebra,  $\mu$  be<br>  $\int$  and  $\beta \in [0,1]$ . filters of a hyper BE-algebra H. Let  $x, y$ **Then Proof.** Assume that  $\mu$  and  $\nu$  are fuzzy weak hyper  $f(x)$  and  $f(x)$ **Theorem 3.10** inters of a hyper BE-algebra  $H$ . Let and  $\beta \in [0,1]$ .  $\overline{\phantom{a}}$ **Proof.** Assume that  $\mu$  and  $\nu$  are fuzzy weak hyper ters of a hyper BE-algebra  $H$ . Let  $x, y \in H$ . Then Assume that  $\mu$  and  $\nu$  are fuzzy weak here. **Theorem 3.10** If  $\frac{1}{2}$  is  $\frac{1}{2}$  if  $\frac{1}{2}$  and  $\frac{1}{2}$  if  $\frac{1}{2}$  if  $\frac{1}{2}$  and  $\frac{1}{2}$  if  $\frac{1}{2$  $\sum_{i=1}^{n}$ 

Slation of 
$$
\mu
$$
,

\n
$$
(\mu \cap \nu)(1) = \min{\mu(1), \nu(1)}
$$
\n
$$
\geq \min{\mu(x), \nu(x)} = (\mu \cap \nu)(x)
$$
\nhyper filter

\nand

and and  $f_{\rm eff}$  and  $f_{\rm eff}$  and  $f_{\rm eff}$  , then  $f_{\rm eff}$  and  $f_{\rm eff}$  a and a shi

$$
\begin{aligned}\n\text{back hyper filter} & \text{(}\mu \cap \nu)(x) = \min\{\mu(x), \nu(x)\} \\
&\in \mu_{\beta,\alpha}^{MT}(1) = \qquad \qquad \geq \min\{\min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\}, \\
&\in \min\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}\} \\
&= \min\{\inf_{z \in y \circ x} \{\min\{\mu(z), \nu(z)\}\}, \\
&\mapsto \alpha \\
&= \min\{\inf_{z \in y \circ x} (\mu \cap \nu)(z), (\mu \cap \nu)(y)\} \\
&\in \mu(\nu) + \alpha\n\end{aligned}
$$

 $z ∈ y ∘ x$ <br>  $\{A\}$  Hence, μ ∩ ν is a fuzzy weak hyper filter of *H*. and  $\overline{\text{max}}$  we  $(\beta \mu(y) + \alpha)$  Hence,  $\mu \cap v$  is a fuzzy weak hyper filter of H.

Theorem 3.11 If  $\mu$  and  $\nu$  are fuzzy weak hyper order the set of a hyper BE-algebra *H* such that  $\mu \subseteq \nu$  or  $\nu \subseteq \mu$ , of H. Then  $\mu \cup v$  is a fuzzy weak hyper filter of H.<br>a fuzzy weak  $\vdots$   $\boldsymbol{\Pi}$ �∈�∘�  $m_{\rm H}$  must so that in the model of  $\sim$ of H. filters of a hyper BE-algebra H such that  $\mu \subseteq v$  or  $v \subseteq \mu$ , �∈�∘�  $\mathbf{r}$  or  $H$ . Figure  $\mu \cup \nu$  is a fuzzy weak rigper filler or  $\pi$ .

**Proof.** Assume that  $\mu$  and  $\nu$  and are fuzzy weak hyper filters of a hyper BE-algebra *H* such that µ ⊆ *v* or �∈�∘�  $v \subseteq \mu$ . Let *x*,  $y \in H$ . Then myper inters of a hyper  $DE - c$ hvner filters of a hvner  $BF<sup>o</sup>$  $\mathbf{F}$  for  $\mathbf{F}$  and  $\mathbf{F}$  and  $\mathbf{F}$  and  $\mathbf{F}$  are fuzzy weak  $+\alpha =$  **Proof.** Assume that  $\mu$  $(1) + u =$  hyper filters of a hyper BE-algebra H such the  $\mathcal{L}$  is a fuzzy weak hypers filter of .  $\nu \subseteq \mu$ . Let  $x, y \in H$ . Then fuzzy weak **Proof.** Assume that  $\mu$  and  $\nu$  and are fuzzy weak <br>(1)  $\mu \alpha$  – **Theorem 3.10** is a fuzzy weak **hyperfuller of a hyper filter of a hyper filter of a hyper filter of**  $\beta\mu(1) + \alpha =$ **ETOOT.** Assume that  $\mu$  and  $\nu$  and are fuzzy weak<br>hyper filters of a hyper BE-algebra H such that  $\mu \subset \nu$  or  $T_{\text{tot}}$  is  $\mu \leq r$  or  $V \subseteq \mu$ . Let  $\lambda, \gamma \in H$ . Then  $\subseteq \mu$ .

$$
(\mu \cup \nu)(1) = \max{\mu(1), \nu(1)}
$$
  
\n
$$
\geq \max{\mu(x), \nu(x)} = (\mu \cup \nu)(x).
$$

Now,  $\mathcal{L}$ 

$$
(\mu \cup \nu)(x) = \max{\mu(x), \nu(x)}
$$
  
\n
$$
\geq \max{\min{\inf_{z \in y \circ x} \mu(z), \mu(y)}}.
$$
  
\n
$$
\min{\inf_{z \in y \circ x} \nu(z), \nu(y)}
$$
  
\n
$$
= \min{\max{\inf_{z \in y \circ x} \mu(z), \mu(y)}}
$$
  
\n
$$
\max{\inf_{z \in y \circ x} \nu(z), \nu(y)}
$$
  
\n
$$
= \min{\inf_{z \in y \circ x} {\max{\mu(z), \nu(z)}}}
$$
  
\n
$$
\max{\mu(y), \nu(y)}
$$
  
\n
$$
= \min{\inf_{z \in y \circ x} (\mu \cup \nu)(z), (\mu \cup \nu)(y)}
$$
.  
\nIn general,  $\max{\min{\min{\max{\mu(y), \nu(y)}}}$ .

In general,  $\max{\min\{\}\min\{\max\{\}\}.$  Suppose  $\frac{1}{2}$  in general,  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  In general,  $\max{\min{\}}$ min $\max{\}$ ). Suppo<br>for this case s case for this case  $\mathfrak m$  general,  $\max_{\{1,1\}}$ 

for this case<br>may  $\mathbf{S}$ 

for this case  
\n
$$
\max\{\min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\},\
$$
\n
$$
\min\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}\}\
$$
\n
$$
\neq \min\{\max\{\inf_{z \in y \circ x} \mu(z), \mu(y)\},\
$$
\n
$$
\max\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}\}.
$$
\nThen there exists  $\alpha \in [0,1]$  such that  
\n
$$
\max\{\min\{\inf_{z \in y \circ x} \mu(z), \mu(y)\},\
$$
\n
$$
\min\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}\}\
$$
\n
$$
< \alpha < \min\{\max\{\inf_{z \in y \circ x} \mu(z), \mu(y)\},\
$$
\n
$$
\max\{\inf_{z \in y \circ x} \nu(z), \nu(y)\}\}.
$$

Thus,  $\alpha < \min \left\{ \inf_{z \in y \circ x} \mu(z), \mu(y) \right\}$ . On the other  $\lim_{z \to \infty} \left( \frac{z \in y \circ x}{z \in y \circ x} \right)$  (2) which is completed the proof. . On the other hand, Thus,  $\alpha < \min \{ \inf \mu(z), \mu(y) \}$ . On the hand,  $\min_{z \in y \circ x} \mu(z), \mu(y)$  <  $\alpha$ , which is a contradiction. This completes the proof. , which is a contradiction. This completes the proof. us,  $\alpha < \min \{\inf \mu(z), \mu(y)\}$ . On the algel ,<br> ompletes the proof.  $T_1$  or  $\zeta$  min  $\Big(\inf_{x \in \zeta} u(x), u(x)\Big)$ other hand,  $\min \left\{ \inf_{\gamma \in \mathcal{N}^{\alpha}} \mu(z), \mu(y) \right\} < \alpha$ , which contradiction. This completes the proof. Thus  $\alpha \leq \min \left\{ \inf_{u(x)} u(y) \right\}$ other hand, min  $\begin{cases} \n\text{if } u(z), u(y) \leq \alpha, \text{ with } 0 \leq \alpha. \n\end{cases}$ contradiction. This completes the proof. Thus  $\alpha \leq \min_{\alpha} \int \inf_{u(x)} u(y) dx$ other hand,  $\min\left\{\inf_{z\in y\circ x}\mu(z), \mu(y)\right\}<\alpha$ , where contradiction. This completes the proof. Thus,  $\alpha < \min\{\min\{\mu(z), \mu(y)\}\}\$ . On the ction. This completes the proof. Thus,  $\alpha < \min \left\{ \inf_{z \in y \circ x} \mu(z), \mu(y) \right\}$ . On the mand,  $\min_{\{z \in \mathcal{Y} \times \mathcal{X}\}} \mu(y)$   $\leq \alpha$ , which is a diction. This completes the proof. Thus,  $\alpha < \min \{ \inf u(z), u(y) \}$ . On the hand,  $\min \left\{ \inf_{z \in y \circ x} \mu(z), \mu(y) \right\} < \alpha$ , which is a doction. This completes the proof.

�∈�∘�

Then, we have the following corollary.<br>filters Then we have the following corollary

**Corollary 3.12** Let  $\{\mu_i : i \in \Lambda\}$  be a nonempty fuzzy weak hyper filters of a hyper **Corollary 3.12** Corollary 3.122 Beta non-matrice and  $R$  and  $R$  and  $R$  and  $R$ of a family of fuzzy weak hyper filters of a hyper filters of  $\alpha$ Then, we have the following corollary. We have the following corollary  $\mathcal{L}$ set of a family of fuzzy weak hyper lifers of a hyper<br>BE-algebra  $H$ , where  $\Lambda$  is an arbitrary indexed set. Then the following statements hold: set of a family of fuzzy weak hyper filters of a hyper with  $\frac{1}{2}$  with ⋃ or fuzzy weak hyper filters of a hype v of fuzzy weak hyper filters of a hy gebra  $H$ , where  $\Lambda$  is an arbitrary indexed set. the following statements hold:

 $\sum_{i \in \Lambda} \sum_{j=1}^{n}$  is an arbitrary indexed set.  $\mathbf{F}$  is the following statement of  $\mathbf{F}$ (i)  $\bigcap_{i \in \Lambda} \mu_j$  is a fuzzy weak hyper filter of H; (ii) if  $\mu_i \subseteq \mu_j$  or  $\mu_j \subseteq \mu_i$  for all  $i, j \in \Lambda$ , then fit  $\bigcap_{i\in\Lambda}\mu_j$  is a fuzzy weak hyper filter of H. (i)  $\bigcap_{i\in\Lambda}\mu_j$  is a fuzzy weak hyper filter of H; (ii) if  $\mu_i \subseteq \mu_j$  or  $\mu_j \subseteq \mu_i$  for a

 $i \in \Lambda$ .<br>Next, we denote by  $FHF(H)$  the set of all fuzzy er filters of a hyper BE-algebra  $H$ . By Corollary obtain the following theorem.  $\frac{1}{2}$  if  $\frac{1}{2}$  for  $\frac{1}{2}$  fo 3.12, we obtain the following theorem. weak hyper filters of a hyper BE-algebra *H*. By Corollary<br>2.42 we obtain the following theorem (per litters or a riyper  $B = \text{arg}$  and  $B$ . By Corollary  $\ddot{\phantom{a}}$ Is a fuzzy weak hyper filter of  $H$ .<br>Next, we denote by  $E I E \langle II \rangle$  the est of ell.  $i$ ivext, we denote by  $PID(II)$  the set or all then hyper filters of a hyper BE-algebra  $H$ . By Corolla

**Theorem 3.13** Let *<sup>H</sup>*be a hyper BE-algebra and  $(fHF(H); \subseteq)$  be a totally ordered set by the set inclusion. Then *(FHF(H);*⊆,∨,∧) is a complete lattice, where  $\tau$  be a total  $\Omega$  13 Let  $U$  be a hyper BE-algebra and  $\tau$ 

$$
\wedge \{ \mu_i \in FHF(H) : i \in \wedge \} = \bigcap_{i \in \Lambda} \mu_i,
$$
  

$$
\vee \{ \mu_i \in FHF(H) : i \in \wedge \} = \bigcap_{i \in \Lambda} \mu_i.
$$

 $(FHF(H); \subseteq)$  be a totally ordered set. Then  $\mu \cap (\nu \cup \lambda)$ =  $U(\mu \cap \lambda)$  and  $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap (\mu \cup \lambda)$ , **Theorem 3.14** Let *H* be a hyper BE-algebra and **COVECT** and  $\overline{C}$  $(\mu \cap \nu) \cup (\mu \cap \lambda)$  and  $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap (\mu \cup \lambda)$ , for all  $\mu, \nu, \lambda \in FHF(H)$ . **Lemma 3.14** Let *H* be a hyper BE-algebra and **CH** ⊏  $\left(\frac{1}{2}\right)^{2}$  ( $\mu$  a) dia  $\mu$   $\in$  ( $\mu$  a) the set  $\mu$  the set  $\mu$  $HF(H): \subseteq$ ) be a totally ordered set. Then  $\mu \cap (\nu \cup \lambda) =$ 

 $\mu, \nu, \lambda \in H H^1(H)$ .

**Proof.** Let  $\mu, \nu, \lambda \in I$ **Proof.** Let  $\mu, v, \lambda \in FHF(H)$  and  $x \in H$ . The  $(x)$  $L(x)$   $(x)$ **Proof.** Let  $\mu$ ,  $\nu$ ,  $\lambda \in FHF(H)$  and  $x \in H$ . Then  $(\mu \cap (\nu \cup \lambda))$   $(x)$  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Proof. Let  $\mu, \nu, \dot{\nu}$  $\mathcal{L}(\mathcal{O})$  $(x)$ 

 $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\nu\{u(x), \text{max}\{v(x), \lambda(x)\}\}\n$  $= max\{min\{\mu(x), \nu(x)\}, min\{\mu(x), \lambda(x)\}\}\$  $= max\{(\mu \cap v)(x), (\mu \cap \lambda)(x)\}$  $= min{\mu(x), (\nu \cup \lambda)(x)}$  $=min{\mu(x), max{\nu(x), \lambda(x)}}$ =  $max{min{ \mu(x), \nu(x), min{ \mu(x), \lambda(x) } } }$  $= max\{(\mu \cap \nu)(x), (\mu \cap \lambda)(x)\}\$ � ∩ ( ∪ )�()  $= max{(\mu \cap v)(x), (\mu \cap \lambda)(x)}$  $\frac{1}{\sqrt{2}}$ � ∩ ( ∪ )�() ⋁{� ∈ ℱℋℱ() ∶ ∈ Λ} = ⋃  $L$  mar<sup>f</sup> min<sup>f</sup>  $\mu(r)$   $\nu(r)$  min<sup>f</sup>  $\mu(r)$   $\lambda(r)$  $\equiv max\{(\mu(y)(x), (\mu(y))(x)\})$  $\mathcal{L} = min\{\mu(x), max\{v(x), \lambda(x)\}\}\$  $-max\{mn\}$  $\mu(x)$ ,  $\nu(x)$   $\gamma$ ,  $mn\{ \mu(x)$ ,  $\nu(x)$   $\}$   $\gamma$  $= max_{\{ \mu \nu \}} (x), (\mu \nu)(x)$ 

 $= ((\mu \cap \nu) \cup (\mu \cup \lambda))(x).$  $= ((\mu \cap \nu) \cup (\mu \cup \lambda))(x).$  $=(({\cup} \cap v) \cup ({\cup} \lambda))({x}).$ 

Hence,  $\mu \cap (\nu \cup \lambda) = (\mu \cap \nu) \cup (\mu \cap \lambda)$ .  $\mathsf{over}$  that  $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap$  $(\mu_0, \nu) \in (\mu_0, \nu)$ ,  $(\nu_1, \nu_2) \in (\mu_1, \nu_1)$  $f(x) = f(x) - f(x)$ <br>a prove that  $f(x) = f(x)$ Hence,  $\mu \cap (\nu \cup \lambda) = (\mu \cap \nu) \cup (\mu \cap \lambda)$ . Similarly, we can prove that  $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap (\mu \cup \lambda)$ .  $\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1}{\sqrt{1-\frac{1$  $\lim_{\mu \to \infty} \mu \in (n - \mu)$  $Hence, \Pi \cap (\nu \cup \lambda) = (\Pi \cap \nu) \cup (\Pi \cap \lambda)$ for all  $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu \cap \lambda)$  $= ((\mu \nu) \circ (\mu \circ \kappa))^{(\lambda)}$ .

From Lemma 3.14, we have the following **Proof.** Let us a set of  $\frac{1}{2}$  and  $\frac{1}{2}$  we have the follow theorem. From Lemma 3.14, we have the following  $\overline{a}$  (neorem. meorem. **Profit Letting 0.11, we have the following** From Lemma 3.14, we have the follow  $\overline{O}$  (  $\overline{O}$  ) $\overline{O}$ 

 $\overline{ }$  integrent 3.15 Let be a hyper- $\alpha$  ordered set. Then is a distribution Theorem  $3.15$  Let be a hyper BE-al be a totally ordered set. Then is a distributed  $\frac{1}{\sqrt{2}}$  $=$  min $($   $=$   $)$ Theorem 3.15 Let be a hyper BE-algebra and  $\blacksquare$  matrice. reform 3.15 Let be a hyper BE-algebra and<br>be a totally ordered set. Then is a distributive complete theorem.  $\mathsf{attice}$ . lattice.  $\frac{1}{\sqrt{2}}$  $=$  max $=$  min $=$   $\frac{1}{\sqrt{2}}$ , min $\frac{1}{\sqrt{2}}$ , min $\frac{1}{\sqrt{2}}$ , min $\frac{1}{\sqrt{2}}$ , min $\frac{1}{\sqrt{2}}$ = maximal contracts and mink of a minimizes comp<br>()  $=$   $\frac{1}{\sqrt{2}}$ ,  $\frac{1}{\$ 

Next, we characterize Noetherian hyper BEthe algebras and Artinian hyper BE-algebras using their fuzzy  $\overline{a}$  weak hyper filters.  $=$   $\frac{1}{\sqrt{2}}$  ,  $\frac{1}{\$  = max{( ∩ )(), ( ∩ )()}  $\frac{1}{2}$ Hence, ∩ ( ∪ ) = ( ∩ ) ∪ ( ∩ ). Next, we characterize N ebras and Artinian nyper BE-algebras using the

A hyper BE-algebra  $H$  is c  $\sim$  the ascending chain condition on weak hyper From  $F \subset F$ similar<br>Album an PE almalana 33.14, we have that have the following the following N nyper BE-algebra *H* is called *Noetherlan* if *I*<br>Satisfies the ascending chain condition on weak hype filters, that is, for any weak hyper filters  $F_1, F_2, F_3, \dots$  of H, theorem.<br>  $x_1$  with  $F_1 ⊆ F_2 ⊆ F_3 ⊆ … ⊆ F_1 ⊆ …$ A hyper BE-algebra *H* is called *Noetherian* if *H*<br>
Similarly, we can prove that prove that satisfies the ascending chain condition on weak hyper  $\mathbb{R}$  in each model.<br>A hyper BE-algebra  $H$ s, for any weak hyper filters  $F_1, F_2, F_3, \dots$  of  $H$ , h  $F_1 \subseteq F_2$ 

There with  $F_i \subseteq F_2 \subseteq F_3 \subseteq ... \subseteq F_i \subseteq ...$ <br>
set. There exists  $n \in \mathbb{N}$  such that  $F_i = F_i + I$  for all  $i \ge$  $\mathfrak{n}.$ A hyper BE-algebra is called *Noetherian*  $n.$ There exists  $n \in \mathbb{N}$  such that  $F_i = F_i + I$  for all  $i \ge$ 

*n*.

A hyper BE-algebra  $H$  is called Artinian if  $H$ s the descending chain condition on weak hyper ulat is,<br> $- E$ (The a total ordered in the set of satisfies the descending chain condition on weak hyper then filters, that is, for any weak hyper filters  $F_i, F_j, F_j, \ldots$  of  $H_i$ with  $F_j \subseteq F_2 \subseteq F_j \subseteq \dots \subseteq F_i \subseteq \dots$  $H$ ; A hyper BE-algebra  $H$  is called Artinian if  $H$ filters, that is, for any weak hyper filters  $F_1, F_2, F_3, \ldots$  of  $H$ , ( $\overline{A}$  hyper BE-algebra  $H$  is called Artinian if  $H$ ers, tha  $\subseteq$   $F_j \subseteq \ldots \subseteq F_i \subseteq \ldots$ 

 $\sum_{n=1}^{\infty}$  characterize  $\sum_{n=1}^{\infty}$  characterize  $\sum_{n=1}^{\infty}$  and  $\sum_{n=1}^{\infty}$ BE-algebras and Artinian hyper BE-algebras using  $\begin{aligned} \mathsf{IZzy} \qquad \qquad & \mathsf{There\ exists\ n \in \mathbb{N} \ such \ that \ } F_i = F_i + I \ \text{for all} \ i \geq 0 \end{aligned}$  $\frac{1}{n}$ . *n*. There exists  $n \in \mathbb{N}$  such that  $F_i = F_i + I$  for all  $i \geq$ 

 $\mathbb{R}^n$  the set of of  $\mathbb{R}^n$  the set of  $\mathbb{R}^n$ Theorem 3.16 Let  $H$  be a hyper BE-algebra. herian if and only if for every fuzzy weak A hyper BE-algebra is called *Noetherian* hyper filters, that is, for any weak hyper filters . By Corollary 3.12, we obtain the following  $\mathcal{L}$ Then *H* is Noetherian if and only if for every fuzzy weak Next, we denote by  $\mathcal{N}$  the set of of  $\mathcal{N}$  $\sum_{n=1}^{\infty}$  Theorem 2.16 Let  $H$  k **EXECTED BE-algebra BE-algebra 1**<br>Noetherian if and only if for every fuzzy weak if  $\alpha$  satisfies the ascending chain condition on  $\alpha$ 

 $\sqrt{a}$  for every function  $\sqrt{a}$ 

hyper filter  $\mu$  of H, the set  $Im(\mu) = {\mu(x):x \in H}$  is a well-<br>(ii)  $\Rightarrow$  (i): Assume ordered subset of  $[0,1]$ .  $0.1$ ].

**Proof.** Assume that H is Noetherian. Suppose that there exists a fuzzy weak hyper filter  $\mu$  of  $H$  such  $\mu$  =  $\mu$  = that Im( $\mu$ ) is not a well-ordered subset of [0,1]. Then there  $F_i \subset F$ exists a strictly infinite decreasing sequence  $\{t_n\}_{n=1}^\infty$  fuzzy set  $\mu$  of H by such that  $\mu(x_n) = t_n$  for some  $x_n \in H$ . Let  $I_n = U(\mu; t_n) = \{x \in H:$ for all new Moreover,  $I_1 \subset I_2 \subset I_3 \subset ...$  is a strictly infinite<br>for all new Moreover,  $I_1 \subset I_2 \subset I_3 \subset ...$  is a strictly infinite ascending chain of weak hyper filters of *H*. This is a contradiction that *H* is Noetherian. Therefore,  $Im(\mu)$  is a where  $F_{\theta} = \emptyset$ . By The  $\mu(x) \ge t_n$ . By Theorem 3.3,  $I_n$  is a weak hyper filter of H,<br>for all a s<sup>30</sup>. Margavax  $I = I = I$  is a strictly infinite well-ordered subset of  $[0,1]$ , for each fuzzy weak hyper filter  $\mu$  of  $H$ . where  $\mathbf{r}$  is  $\mathbf{r}$  is the set Im()  $\mathbf{r}$  in  $\mathbf{r}$  is the set Im()  $\mathbf{r}$  in  $\mathbf{r}$  is the set Im()  $\mathbf{r}$  $\sin$  that  $H$  is Noetherian Suppose such the fuzzy weak hyper filter  $\mu$  of  $H$  suc  $\alpha$  contradiction that *H* is Noetherian. Therefore,  $Im(\mu)$  is a for  $\frac{1}{2}$  of  $\begin{bmatrix} 0 & 1 \end{bmatrix}$  for each fuzzy weak dered subset of  $[0,1]$ , for each fi  $T_{\text{m}}$  exists  $\mu$  or  $H$ . **Corollary 3.17** Let  $\mathcal{L}$  **Corollary 3.17** Let  $\mathcal{L}$  for a hyper BE-algebra. If  $\mathcal{L}$ 

Then there exists a strictly infinite ascending chain equivalent:  $F_1 \subset F_2 \subset F_3 \subset \ldots \subset F_n \subset \ldots$  of weak hyper filters of *H*. We  $\qquad \qquad$  (i) *H* is Artinian; Conversely, assume that for every fuzzy weak hyper filter  $\mu$  of *H*, the set  $Im(\mu) = {\mu(x): x \in H}$  is a  $T = {t_1, t_2, ...\}\cup {0}$ well-ordered subset of . Suppose that is not Noetherian. define the fuzzy weak hyper filter of  $\mu$  of  $H$  by  $\ddot{\phantom{0}}$  $\ddot{\phantom{a}}$  $\alpha$   $\alpha$   $\beta$   $\beta$   $\beta$  $f_{\rm eff}$  each  $\epsilon$ y weak  $\overline{T}$  .  $F_1 \subseteq F_2 \subseteq F_3 \subseteq ... \subseteq F_n \subseteq ...$  or weak nyper niters or  $\frac{1}{2}$  is a well-ordered subset of  $\frac{1}{2}$ . in **c**quivalents:

$$
\mu(x) = \begin{cases} 0 & \text{if } x \notin F_n \\ \frac{1}{n} & \text{if } x \in F_n - F_{n-1} \text{ for } n = 1, 2, \dots; \end{cases}
$$

where  $F_0 = \emptyset$ . By Theorem 3.6, μ is a fuzzy<br>Supper weak hyper filter of  $H$ , but  $Im(\mu)$  is not a well-ordered  $\sim$  $\overline{a}$ subset of [0,1]. We get a contradiction. Consequently,  $H = \frac{W}{U}$ is Noetherian. **Corollary 3.17 Corollary 3.17**  $\Box$  is Noetherian.

**Corollary 3.17** Let *H* be a hyper BE-algebra. If for every fuzzy weak hyper filter  $\mu$  of  $H$  such that  $\text{Im}(\mu)$  $\frac{1}{2}$  is a finite set, then *H* is Noetherian. **Euronary 3.17** Let *H* be a hyper BE-aigebra. If  $\mu$  of *H*, which is a contradiction that  $\mu$  for every fuzzy weak hyper filter  $\mu$  of *H* such that  $\text{Im}(\mu)$ 

**Theorem 3.18** Let  $H$  be a hyper BE-algebra and  $T = \{t_1, t_2, ...\} \cup \{0\}$ , where  $\{t_n\}_{n=1}^{\infty}$  is a strictly decreasing in  $[0,1]$ . Then the following conditions are equivalent:

 $(i)$  *H* is Noetherian;

(ii) for every fuzzy weak hyper filter  $\mu$  of *H*,  $f \log \theta$ if Im( $\mu$ )  $\subseteq$  T, then there exists  $k \in \mathbb{N}$  such that  $Im(\mu) \subseteq \{t_1, t_2, \ldots, t_k\}$   $\mu(x) = \begin{cases} 0 & \text{if } x \notin F_1, t_k \\ t_n & \text{if } x \in F_n - F_{n+1} \\ 1 & \text{if } x \in F_n \end{cases}$ *t*<sub>2</sub>, ..., *t*<sub>k</sub>}∪{0}.  $\mathcal{P}$  $\mu$  of  $H$ ,

**Proof.** (i)  $\Rightarrow$  (ii): Assume that *H* is a Noetherian. Let  $\mu$  be a fuzzy weak hyper filter of  $H$  such that  $\text{Im}(\mu)$  $\subseteq$  T. By Theorem 3.16,  $\text{Im}(\mu)$  is a well-ordered subset of [0,1]. Hence, there exists  $k \in \mathbb{N}$  such that  $Im(\mu) \subseteq \{t_1, t_2, \ldots, t_n\}$  $\mathbb{R}^2$  . By Theorem 3.16, Implies we have the set of  $\mathbb{R}^2$ *..., t<sub>k</sub>*}∪{0}.  $\ldots, \ell_k$   $\cup$   $\cup$   $\cup$   $\cdot$  $\mathbf{a}$  Noetherian  $\overline{t}$  $n(\mu) \subseteq \{t_1, t_2\}$ subset  $\overline{I}$ 

 $\text{Lipper}$  and  $\mu$  or  $H$ ,  $\mu$  mapping  $\mu$  or  $H$ ,  $\mu$  mapping  $\mu$  or  $H$ ,  $\mu$  and  $\mu$  and  $\mu$  are  $\mu$  and  $\mu$  are  $\mu$  and  $\mu$  are  $\mu$  ar original, suppose the continuum  $\mu$   $\equiv$   $\sigma$ <sub>*p*</sub>,  $\sigma$ <sub>*p*</sub>,  $\sigma$ <sub>*c*</sup><sub>*p*</sub>,  $\sigma$ <sub>*c*</sup><sub>*p*</sub>,  $\sigma$ <sup>2</sup>  $\sigma$ <sub>*c*</sub>  $\sigma$ <sup>2</sup>  $\sigma$ of [0,1]. Then there  $F_1 \subset F_2 \subset F_3 \subset ...$  of weak hyper filters of H. We define a (ii)  $\Rightarrow$  (i): Assume that for every fuzzy weak hyper filter  $\mu$  of *H*, if Im( $\mu$ )  $\subseteq$  T, then there exists  $k \in \mathbb{N}$  $\int_1$  fuzzy set  $\mu$  of *H* by

$$
H:
$$
  
\n
$$
\mu(x) = \begin{cases} 0 & \text{if } x \notin F_n \\ t_n & \text{if } x \in F_n - F_{n-1} \text{ for } n = 1, 2, \dots; \end{cases}
$$
  
\n
$$
\mu(x) = \begin{cases} 0 & \text{if } x \in F_n - F_{n-1} \text{ for } n = 1, 2, \dots; \end{cases}
$$

Tuzzy weak hyper<br>tion. Therefore,  $H$  is Noetherian. lore,  $Im(\mu)$  is a writter  $\int_{0}^{\infty}$ . By medicing 5.0,  $\mu$  is a razzy weak<br>zy weak hyper hyper filter of H. This is a contradiction with our assumpa<br>
every  $F_0 = \emptyset$ . By Theorem 3.6,  $\mu$  is a fuzzy weak<br>
every filter of  $H_0 = \emptyset$ . By Theorem 3.6,  $\mu$  is a fuzzy weak  $\mathbf{F} - \varnothing$  By Theorem 3.6  $\mu$  is a fuzzy weak

 $\frac{1}{2}$ <br>Theorem 3.19 Let  $H$  be a hyper BE-algebra and r every fuzzy weak<br>= { $\mu(x): x \in H$ } is a<br> $T=\{t_1, t_2, ...\} \cup \{0\}$ , where  $\{t_n\}_{n=1}^{\infty}$  is a strictly increasing  $\frac{k}{n+1}$  is a<br>loetherian. sequence in [0,1]. Then the following conditions are اب<br>ا if if  $T_{\text{heat}}$ ∪,⊥ j. ⊥⊔icii ur  $=$   $\frac{1}{1}$  io a calcay ...  $f_1$  is a<br>sequence in [0,1]. Then the following conditions are<br> $f_1$  sequence in [0,1]. Then the following conditions are equivalent:  $\Box$ Fry fuzzy weak **Theorem 3.19** Let  $H$  be a hyper BE-algebra and where  $\{t_n\}_{n=1}^{\infty}$  is a strictly increasing Hence, is a fuzzy weak hyper filter of . We

# $H$ . We (i) *H* is Artinian;

if  $\text{Im}(\mu) \subseteq T$ , then there exists  $k \in \mathbb{N}$  such that  $\text{Im}(\mu) \subseteq$ for each  $n \in \mathbb{N}$ ;  $\{t_1, t_2, ..., t_k\} \cup \{0\}$ . (ii) for every fuzzy weak hyper filter  $\mu$  of  $H$ ,  $\frac{1}{2}$  is  $\frac{1}{2}$  $H$  by  $(ii)$  for every fuzzy weak hyper filter  $\mu$  of  $H$ , Iter  $\mu$  of  $H$ ,  $\cup$ {0}.

for  $n = 1, 2, ...$ ;<br>**Proof.** (i)  $\Rightarrow$  (ii): Assume that *H* is Artinian. Let  $U(\mu; t_{i_m})$  for  $m=1, 2, ...$  Th g sequence on ∘<br>∖for sta  $\mu$  be a fuzzy weak hyper filter of H such that Im( $\mu$ )  $\subseteq$  T.<br>a fuzzy  $\frac{1}{2}$ The such that the such that  $\lim_{m \to \infty} \frac{1}{m}$  is a fuzzy suppose that  $t_{i_1} < t_{i_2} < \cdots < t_{i_m} < \dots$  is a strictly a well-ordered increasing aggregate of almost in  $\lim_{m \to \infty}$  is a strictly bisequently,  $H$   $U(\mu; t_{i_m})$  for  $m=1,2,...$  This implies that  $I_1 \supset I_2 \supset ... \supset I_m$  $\cup$ ( $\mu$ ,  $\iota_{lm}$ , is  $m=1,2,...$  This implies that  $I_1 \supseteq I_2 \supseteq ... \supseteq I_m$ <br> $\supseteq ...$  is a strictly descending chain of weak hyper filters gebra. If  $\mu$  of *H*, which is a contradiction that *H* is Artinian. **Proof.** (i)  $\sum_{i=1}^{n}$  (ii) Assume that  $\sum_{i=1}^{n}$ a well-ordered<br>increasing sequence of elements in  $\text{Im}(\mu)$ . Let  $I_m =$ **Corollary 3.20** Let be a hyper BE-algebra. If for **Proof.** (i)  $\Rightarrow$  (ii): Assume that H is Artinian. Let ⇒ ... is a strictly descending chain of weak hyper filters  $\frac{1}{2}$ nat  $t_{i_1} < t_{i_2} < \cdots < t_{i_m} < \dots$  is a strictly sequence of elements in  $\text{Im}(\mu)$ . Let  $I_m =$ 

at Im( $\mu$ )  $(ii) \Rightarrow (i)$ : Assume that for every fuzzy weak hyper filter  $\mu$  of  $H$ , if  $\text{Im}(\mu) \subseteq I$ , then there exists ctly decreasing not Artinian. Then there exists a strictly descending chain equivalent:  $F_1 \supset F_2 \supset \dots \supset F_n \supset \dots$  of weak hyper filters of *H*. We define a fuzzy set  $\mu$  in  $H$  by hyper filter  $\mu$  of H, if Im( $\mu$ )  $\subseteq$  T, then there exists  $k \in \mathbb{N}$ a and such that  $\text{Im}(\mu) \subseteq \{t_1, t_2, ..., t_k\} \cup \{0\}$ . Suppose that *H* is  $\mathsf{EXISIS}\ K \in \mathbb{N}$ for near  $\epsilon$  is a distribution of  $\epsilon$  $\frac{1}{2}$  is not Artinian. Then the exists and the exists are existence exists and  $\frac{1}{2}$  are existence exists and  $\frac{1}{2}$  are existence of  $\frac{1}{2}$  and  $\frac{1}{2}$  are exists and  $\frac{1}{2}$  are exists and  $\frac{1}{2}$  ar hyper BE-algebras and Artinian hyper BE-algebras

er inter 
$$
\mu
$$
 or  $H$ ,  
\n
$$
\mu(x) = \begin{cases}\n0 & \text{if } x \notin F_1, \\
t_n & \text{if } x \in F_n - F_{n+1} \\
1 & \text{if } x \in F_n\n\end{cases}
$$
 for  $n = 1, 2, ...,$   
\nfor all  $n \in \mathbb{N}$ .

 $x, y \in H$ . Thus, we can divide to be three cases We have that  $\mu(I) = 1 \ge \mu(x)$ , for all  $x \in H$ . Next, let ∉ �, *x*,y∈*H*. Thus, we can divide to be three cases, as follows.

of  $\begin{array}{ll} \textsf{Case 1:} \; x \not\in F, \; \textsf{Then} \; y \mathbin{\circ} x \not\in F, \; \textsf{O} \end{array}$  $h_{2}$ , thus, the contradiction with our contradiction with our contradiction  $\mathcal{L}$ Thus, Case 1:  $x \notin F_1$ . Then  $y \circ x \notin F_1$  or  $y \notin F_1$ . ∈ � − ���

Case 2:  $x \in F_n - F_{n+1}$  for some  $n=1,2,...$ . Then  $y \circ x \nsubseteq F_{n+1}$  or  $y \notin F_{n+1}$ . We obtain that  $\mu(y) \le t_n$  or  $\mu(z)$  ≤  $t_n$  for some  $z \in y \circ x \circ F_{n+1}$ . So,  $\min\{\inf_{z \in y, x} \mu(z), \mu(y)\}$ ≤  $t_{n} = \mu(x)$ ..

Case 3:  $x \in F_n$  for all  $n \in \mathbb{N}$ . Clearly,  $\mu(x) =$  $1 \geq min$  { $\inf_{z \in y.x} μ(z), μ(y)$ }.

Hence, µ is a fuzzy weak hyper filter of *H*. We have a contradiction with our assumption. Consequently, *H* is Artinian.

**Corollary 3.20** Let *H* be a hyper BE-algebra. If for every fuzzy weak hyper filter  $\mu$  of  $H$ , Im( $\mu$ ) is a finite set, then *H* is Artinian.

#### **Conclusions**

The concept of fuzzy weak hyper filters in hyper BE-algebras is introduced and investigated. It was shown that the set of all fuzzy weak hyper filters of hyper BE-algebras is a distributive complete lattice. Also, the concepts of Noetherian hyper BE-algebras and Artinian hyper BE-algebras are characterized by their fuzzy weak hyper filters. In future work, we will study the concept of characterizations of fuzzy weak hyper filters in hyper BE-algebras.

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